

# Recent Trends in Modelling Spatio-Temporal Data

## *Sviluppi recenti nella modellistica dei dati spazio-temporali*

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### **Riassunto.**

Il lavoro fornisce una disamina delle più recenti metodologie proposte nell'ambito dei modelli spazio-temporali. Nel tentativo di proporre una visione unificata delle metodologie trattate, viene fornita prima una descrizione dei vari tipi di dati spazio-temporali. Successivamente, si procede con la discussione dei modelli per processi spazialmente continui. La modellistica spazio-temporale è stata largamente utilizzata per affrontare problemi in ambito ambientale, geostatistico, idrologico e meteorologico. Questo articolo fornisce una analisi dei metodi correntemente applicati in molte di queste aree.

**Keywords:** Bayesian Inference; Gibbs Sampler; Kalman Filter; Kriging; Markov Chain Monte Carlo; Space-Time model.

## **1. Introduction**

In recent years there has been a tremendous growth in the statistical models and techniques to analyze spatio-temporal data such as air-pollution data. Spatio-temporal data arise in many other contexts e.g. disease mapping and economic monitoring of real estate prices. Often the primary interests in analyzing such data are to smooth and predict time evolution of some response variables over a certain spatial domain. Typically, such predictions are made from data observed on a large number of variables which themselves vary over time and space. These spatio-temporal data sets can be very large, for instance, air pollution measurements are often observed every day at over one hundred locations in the UK and the last ten years' data may be available. There are many other important areas where spatio-temporal data are used to detect recognizable and meaningful patterns as well as to make predictions. Examples include hydrology, ecology, geology, social

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sciences, many areas of medicine such as brain imaging, wildlife population monitoring and tracking, and machine vision.

In order to obtain a high degree of accuracy in analysis and predictions of a response variable, such as the amount of air pollution, mathematical models are employed which explicitly include the underlying uncertainty in the data. Such models are statistical in nature and, if appropriately chosen, allow accurate forecasting in future time periods and interpolation over the entire spatial region of interest. In addition, they allow us to estimate the size of possible errors associated with our forecasts. Essential to this process is the use of modern statistical modeling techniques. Such statistical modeling of spatio-temporal data is a challenging task which requires the manipulation of large data sets and the ability to fit realistic and complex models. Often, the required solutions are not available in closed mathematical form and computer intensive methods are needed. In addition, various associated questions have to be addressed: Is the model adequate for the data? Can a better model be found? Can data values that are outliers relative to the model be identified? How accurate are the predictions? Will the model be able to cope with future, possibly more complex, data sets?

The ideas behind the spatio-temporal modeling can be broadly cross-classified according to: (a) their motivation, (b) their underlying objectives and (c) the scale of data. Under (a) the motivations for models can be classified into four classes: (i) extensions of time series methods to space (ii) extension of random field and imaging techniques to time (iii) interaction of time and space methods and (iv) physical models. Under (b) the main objectives can be viewed as either data reduction or prediction. Finally, under (c) the available data might be sparse or dense in time or space respectively, and the modeling approach often takes this scale of data into account. In addition, the data can be either continuously-indexed or discretely-indexed in space and/or time. Based on these considerations, especially (i) – (iii), statistical model building and their implementation take place.

The plan of this review is as follows. In Section 2 we discuss key text books, monographs and review articles. Section 3 discusses three basic spatio-temporal data types. We describe the basic modeling elements and techniques for point reference data in Section 4. The main application areas are listed in Section 5. We conclude with a discussion in Section 6.

## **2. Key references**

### **2.1 Text books and monographs**

There is a huge number of textbooks, edited volumes and monographs discussing modeling and analysis of spatial data. For example, Cressie (1993) discusses various concepts in spatial statistics including kriging using a high level of mathematics, but it does not do hierarchical modeling using modern computing methods. The book by Stein (1999) also provides a theoretical treatise on kriging. There are several texts in theoretical geostatistics, see for example Wackernagel (1998) and Chiles and Delfiner (1999). Schabenberger and Gotway (2004) discuss statistical methods for spatial data analysis. For environmental data monitoring and modeling, deterministic interpolations and conditional stochastic simulation and many other related topics, see for example, the books by Barnett (2004) and Kanevski and Maignan (2004). Some recent texts and monographs in spatial cluster modeling and analysis of spatial point processes include the book by Diggle (2003), the

edited volume by Lawson and Denison (2002), and Møller and Waagepetersen (2003). For modeling and computation using Gaussian Markov random fields see the book by Rue and Held (2005). Christakos (2000) promotes the view that a deeper understanding of a theory of knowledge is an important prerequisite for the development of improved mathematical models of scientific mapping.

The recent text book by Banerjee, Carlin and Gelfand (2004) provides an excellent starting point for researchers in this area. Although the book primarily covers hierarchical modeling and analysis of spatial data with an emphasis towards making Bayesian inference, it discusses spatio-temporal modeling in some detail and discusses a range of topics including multivariate modeling, spatial epidemiology, areal data modeling and many more.

## 2.2 History and review articles

Cressie (1994) and Goodall and Mardia (1994) are among the very early authors who obtained statistical models for spatio-temporal data. In a discussion paper Mardia *et al.* (1998) have introduced a combined approach which they call kriged-Kalman filter (KKF) modeling. Motivated by particular applications, the field has adopted various modeling strategies. There are many monographs and review articles on the related areas of spatial statistics and point process modeling. See for example the edited volume by Mardia *et al.* (1999). Recent thinking in the field has been surveyed by Brown *et al.* (2001), Haslett and Raftery (1989), Kyriakidis and Journel (1999), Kent and Mardia (1994, 2002), Mardia *et al.* (1998), Sahu and Mardia (2005), Stroud *et al.* (2001), Wikle and Cressie (1999), and Wikle *et al.* (1998).

See Mardia (1988) for multivariate conditionally autoregressive models for multivariate random fields and Gelfand *et al.* (2005) for spatio-temporal modeling using dynamic models. In this proceedings Fontanella *et al.* (2005) unify various techniques related to generalized eigenvalue decomposition, and a particular spatio-temporal model is highlighted.

## 3. Types of data

In order to model spatio-temporal data there is an obvious need to keep track of the spatial location, denoted by  $s$  in a region  $D$  say, and the time point,  $t \in \mathbb{R}$  of an observation, say  $Z(s, t)$ . There may be additional covariate information available which we denote by  $x(s, t)$ . Different data types arise by the ways in which the points  $s$  are observed in  $D$ .

Typical *point reference data* arise when  $s$  varies continuously over a fixed study region  $D$ . For example, we may have observed the response  $Z(s, t)$  and the covariates at a set of  $n$  locations denoted by  $s_i$ ,  $i = 1, \dots, n$  and at  $T$  time points so that  $t = 1, \dots, T$ . The set of spatial locations can either be fixed monitoring stations, like in an air pollution example, or can vary with time for example data obtained from a research ship measuring ocean characteristics as it moves about in the ocean. We shall discuss modeling these type of data in some detail in Section 4. We discuss two other important and often used data types in Sections 3.1 and 3.2 before discussing models for point reference data in Section 4.

### 3.1 Areal data

The data are often called *areal or block level data* where the fixed region  $D$  is partitioned into a finite number of areal units with well defined boundaries, e.g. postcodes, counties or districts etc. Here an observation is thought to be associated with an areal unit of non-zero volume rather than a particular location point, e.g. a latitude-longitude pair on the map, see for example the often quoted Scottish lip cancer data (Clayton and Kaldor, 1987) as analyzed by the GeOBugs software (Spiegelhalter *et al.*, 1996) available from `mrc-bsu.cam.ac.uk`.

Typical areal data are represented by a *choropleth map* which uses shades of color or grey scale to classify values into a few broad classes, like a histogram. Such a map provides adjacency information of the areal units (blocks or regions). Some statistical issues here are spatio-temporal smoothing, inference and predictions for new areal units. Gaussian Markov random fields (GMRF) and conditional auto-regressive (CAR) models are appropriate here. See, for example Besag *et al.* (1991) and the book by Rue and Held (2005) and the references therein for more details and many related applications.

Recently there has been a particular interest in the joint spatial analysis of area-specific rates of several potentially related diseases, see for example, Gelfand and Vounatsou (2003), Held *et al.* (2005), Hogan and Tchernis (2004), Knorr-Held and Best (2001), Knorr-Held (2000), Schmid and Held (2004), Sun *et al.* (2000) and Wang and Wall (2003). The purpose here is to detect common spatial patterns in the underlying disease-specific risk surfaces. Models have either been based on a multivariate extension of the widely used CAR model or on spatial generalizations of a factor analysis type model. Best and Tzala (2005) extend the factor analysis approach to consider the joint analysis of spatial-temporal variations in area-specific rates of several diseases over time. They adopt a Bayesian hierarchical modeling framework, and consider various prior formulations for the latent spatial and temporal factors reflecting the shared pattern of risk. The model is motivated by an analysis of age standardized annual mortality rates of 17 cancer sites across the 51 administrative districts in Greece for the years 1981 to 1999.

### 3.2 Point Processes

Spatial point pattern data arise when an event of interest, e.g. outbreak of a disease, occurs at random locations, that is,  $D$  is random and its index set gives the spatial point pattern; the notion of a response variable is not meaningful here, but there can be additional co-variate information at the event locations. See the books by Diggle (2003) and Møller and Waagepetersen (2003) for many examples and theoretical developments.

Spatio-temporal point process data are naturally found in a number of disciplines, including (human or veterinary) epidemiology where extensive data-sets are also becoming more common. One important distinction in practice is between processes defined as a discrete-time sequence of spatial point processes, or as a spatially and temporally continuous point process. Recently Peter Diggle and his coauthors have developed several approaches to the analysis of spatio-temporal point process data, each motivated by a particular application. These include discrete-time modeling, exemplified by annual records of the spatial distribution of bovine tuberculosis cases in Cornwall (Diggle *et al.*, 2005); empirical modeling, exemplified by a log-Gaussian Cox process model for real-time monitoring of gastro-enteric disease in Hampshire (Brix and Diggle, 2001); mechanistic modeling, exemplified by a model published as Keeling *et al.* (2001) for the 2001

UK foot-and-mouth epidemic. Diggle *et al.* (1995) develop a test to determine if there is space-time interaction as a general phenomena in a data set.

## 4. Modeling point reference data

The general aim is to model and analyze the spatio-temporal random variable  $Z(\mathbf{s}, t)$  indexed by spatial coordinates  $\mathbf{s}$  and by time  $t \in \mathbb{R}$ . In most applications  $Z$  will be univariate, the spatial coordinate will be continuous and two dimensional describing the latitude-longitude pair (or its equivalent easting and northing coordinates), and the time coordinate will be discrete and univariate. Often, the response  $Z(\mathbf{s}, t)$  is observed at a number of fixed monitoring sites  $\mathbf{s}_i$ ,  $i = 1, \dots, n$ , say at  $T$  time points so that  $t = 1, \dots, T$ .

Here we discuss some key concepts in modeling spatio-temporal data. See Banerjee, Carlin and Gelfand (2004, Chapter 2) for a fuller description and many exploratory analysis methods.

The modeling strategies begin by assuming the hierarchical structure:

$$Z(\mathbf{s}_i, t) = Y(\mathbf{s}_i, t) + \epsilon(\mathbf{s}_i, t), \quad i = 1, \dots, n, \quad t = 1, \dots, T, \quad (1)$$

where  $Y(\mathbf{s}_i, t)$  is a space-time process and the error term  $\epsilon(\mathbf{s}_i, t)$  is a white noise process and specifically assumed to follow  $N(0, \sigma_\epsilon^2)$  independently. In principle,  $\sigma_\epsilon^2$  could evolve in time but in many applications it can be treated as a constant for the sake of parsimony.

The space-time process  $Y(\mathbf{s}_i, t)$  is expressed as

$$Y(\mathbf{s}_i, t) = \mu(\mathbf{s}_i, t) + w(\mathbf{s}_i, t) \quad (2)$$

where  $\mu(\mathbf{s}_i, t)$  is a mean process driven by observed covariates  $\mathbf{x}(\mathbf{s}_i, t)$  and  $w(\mathbf{s}_i, t)$  is a zero mean spatio-temporal process. A particular covariance structure must be assumed for the  $w(\mathbf{s}_i, t)$  process. The pivotal space-time covariance function is defined as

$$C(\mathbf{s}_1, \mathbf{s}_2; t_1, t_2) = \text{Cov}[w(\mathbf{s}_1, t_1), w(\mathbf{s}_2, t_2)].$$

The zero mean spatio-temporal process  $w(\mathbf{s}, t)$  is said to be *covariance stationary* if

$$C(\mathbf{s}_1, \mathbf{s}_2; t_1, t_2) = C(\mathbf{s}_1 - \mathbf{s}_2; t_1 - t_2) = C(d; \tau),$$

where  $d = \mathbf{s}_1 - \mathbf{s}_2$  and  $\tau = t_1 - t_2$ . The process is said to be *isotropic* if

$$C(d; \tau) = C(\|d\|; |\tau|),$$

that is, the covariance function depends upon the separation vectors only through their lengths  $\|d\|$  and  $|\tau|$ . Processes which are not isotropic are called *anisotropic*. In the literature isotropic processes are popular because of their simplicity and interpretability. Moreover, there is a number of simple parametric forms available to model those.

A further simplifying assumption to make is the assumption of separability see for example, Mardia and Goodall (1993). The process  $w(\mathbf{s}, t)$  is said to be *separable* if

$$C(\|d\|; |\tau|) = C_s(\|d\|) C_t(|\tau|).$$

Now suitable forms for the functions  $C_s(\cdot)$  and  $C_t(\cdot)$  are to be assumed. A very general choice is to adopt the Matèrn covariance function (Matèrn, 1986) given by:

$$C(u) = \sigma^2 \frac{1}{2^{\nu-1} \Gamma(\nu)} (2\sqrt{\nu}u\phi)^\nu K_\nu(2\sqrt{\nu}u\phi), \quad \phi > 0, \nu \geq 1, u > 0 \quad (3)$$

where  $K_\nu(\cdot)$  is the modified Bessel function of second kind and of order  $\nu$ , see e.g. Abramowitz and Stegun (1965, Chapter 9). Popular special cases of the Matèrn family are: (i)  $\nu = 1/2$  corresponding to the exponential model  $C(u) = \sigma^2 \exp(-\phi u)$  and (ii)  $\nu = 3/2$  which leads to  $C(u) = \sigma^2(1 + \phi u) \exp(-\phi u)$  and (iii) Gaussian,  $C(u) = \sigma^2 \exp(-\phi^2 u^2)$  when  $\nu \rightarrow \infty$ .

There is a growing literature on methods for constructing non-separable and non-stationary spatio-temporal covariance functions that are useful for modeling. See for example, Gneiting (2002) develops a class of non-separable covariance functions. A simple example is:

$$C(\|d\|; |\tau|) = (1 + |\tau|)^{-1} \exp \left\{ -\|d\| / (1 + |\tau|)^{\beta/2} \right\},$$

where  $\beta$  is a space-time interaction parameter. There are many other methods, for example Schmidt and O'Hagan (2003) construct non-stationary spatio-temporal covariance structure via deformations. There are other ways to construct non-separable covariance functions, for example by mixing more than one spatio-temporal processes, see e.g. Sahu *et al.* (2004) or by including a further level of hierarchy where the covariance matrix obtained using  $C(\|d\|; |\tau|)$  follows a inverse-Wishart distribution centered around a separable covariance matrix, see e.g. Brown *et al.* (1994) and also Gelfand *et al.* (2004b). Section 8.3 in Banerjee, Carlin and Gelfand (2004) also lists some more strategies.

One well-known non-separable covariance function in Physics is obtained from Taylor's frozen field hypothesis which states that

$$\text{Cov} \{w(0, 0), w(\mathbf{v}t, 0)\} = \text{Cov} \{w(0, 0), w(0, t)\}$$

for some vector  $\mathbf{v}$  (velocity) so that the space vector  $\mathbf{v}$  in the left hand side leads to time component on the right hand side. Wheater *et al.* (2000, p.586, Figure 5) have given one application of this for modeling a rainfall data .

The spectral domain can be used as an efficient and interpretable framework for comparing and contrasting many of these methods, including the deformation approach, kernel convolutions, spatially varying anisotropic kernels and spatially adaptive spectra, see e.g. Fuentes (2002). Pintore and Holmes (2005) review these methods and show how they relate to one another. They also point to generalizations which result in spatially adaptive covariance functions with standard forms, such as the Gaussian or the Matèrn, but now with "localised" parameters.

## 4.1 The Bayesian kriged-Kalman filter model

This model is developed by using what are known as principal kriging functions, see Mardia *et al.* (1998) and Sahu and Mardia (2005) for full details. Here we only provide a brief outline. Suppose that the mean process in (2) is the sum of two processes

$$\mu(\mathbf{s}_i, t) = \sum_{j=1}^p h_{\mathbf{s}_i, j} \alpha_{tj} + \mathbf{x}(\mathbf{s}_i, t)^T \boldsymbol{\beta}_t$$

where the first term is the kriged-Kalman filter term described below and the second term is the regression term with time varying regression parameters  $\beta_t$ .

Let  $H$  be the matrix of order  $n \times p$  with elements  $h_{s_i,j}$ . Assume that the first column of  $H$  to be the unit vector, denoted by  $\mathbf{1}$ . The other columns are obtained as follows: First, obtain

$$B = \Sigma_\gamma^{-1} - \frac{1}{\mathbf{1}'\Sigma_\gamma^{-1}\mathbf{1}}\Sigma_\gamma^{-1}\mathbf{1}\mathbf{1}'\Sigma_\gamma^{-1}.$$

Now perform the spectral decomposition of  $B$ , so that  $B = UEU'$  and  $B\mathbf{u}_i = e_i\mathbf{u}_i$ , where  $U = (\mathbf{u}_1, \dots, \mathbf{u}_n)$  and  $E = \text{diag}(e_1, \dots, e_n)$ , and we assume without loss of generality that the eigenvalues are in non-decreasing order,  $e_1 = 0 < e_{2+1} \leq \dots \leq e_n$ . Finally, the matrix  $H$  is taken as  $H = (\mathbf{1}, e_2\Sigma_\gamma\mathbf{u}_2, \dots, e_p\Sigma_\gamma\mathbf{u}_p)$ . For Kalman filtering Sahu and Mardia (2005) assume the dynamic time series model  $\alpha_t = \alpha_{t-1} + \eta_t$ , where  $\eta_t \sim N(\mathbf{0}, \Sigma_\eta)$ .

## 4.2 A kernel convolution approach

A general class of stationary processes can be built using a kernel convolution approach initially discussed by Ver Hoef and Barry (1998). In a series of papers Higdon and his co-authors has popularized the approach by adopting a discrete version of the approach, see for example Higdon (1988). See Sahu and Challoner (2005) for some recent developments and an interesting application on joint modeling of ocean temperature and salinity.

The spatio-temporal process  $w(\mathbf{s}, t)$ , is thought to be induced by kernel convolution effects of a single latent spatio-temporal process  $v(\omega_l, t_m)$  where  $\omega_l$  denotes a spatial location and  $t_m$  denotes a time point.

Let  $K(d_s, d_t)$  denote the joint kernel in space and time where  $d_s$  and  $d_t$  are the distances in space and in time, respectively. Let  $\omega_l, l = 1, \dots, L$  denote the grid locations where the spatial smoothing kernels will be centered; similarly let  $t_m, m = 1, \dots, M$  denote the equi-spaced time points where the temporal kernels will be centered. Now we write:

$$w(\mathbf{s}, t) = \sum_{l=1}^L \sum_{m=1}^M K(\|\mathbf{s} - \omega_l\|, |t - t_m|) v(\omega_l, t_m)$$

where  $\|\mathbf{s} - \omega_l\|$  denotes the distance between the locations  $\mathbf{s}$  and  $\omega_l$ .

A simple form of

$$K(d_s, d_t) = K_s(d_s) K_t(d_t)$$

where  $K_s(d_s)$  is a kernel in space and  $K_t(d_t)$  is a kernel in time. The kernels  $K_s$  and  $K_t$  can be chosen to be any valid covariance function, for example belonging to the Matérn family (3).

## 5. Application areas

### 5.1 Environmental pollution monitoring

Space-time modeling of pollutants has some history including, e.g., Guttorp *et al.* (1994), Haas (1995) and Carroll *et al.* (1997). In recent years, hierarchical Bayesian approaches for spatial prediction of air pollution have been developed (Brown *et al.*, 1994). Zidek *et al.* (2002) developed predictive distributions on non-monitored PM<sub>10</sub> concentrations (a type of air pollution particles with diameters less than 10 $\mu\text{m}$ ) in Vancouver, CA. They

noted the under-prediction of extreme values in the pollution field, but their methodology provided useful estimates of uncertainties for large values. Cressie *et al.* (1999) compared kriging and Markov-random field models in the prediction of PM<sub>10</sub> concentrations around Pittsburgh, PA. Kibria *et al.* (2000) developed a multivariate spatial prediction methodology in a Bayesian context for the prediction of PM<sub>2.5</sub> in Philadelphia, PA. This approach used both PM<sub>2.5</sub> and PM<sub>10</sub> data at monitoring sites with different start-up times. Shaddick and Wakefield (2002) propose short term space-time modeling for PM<sub>10</sub>.

Smith *et al.* (2003) propose a spatio-temporal model for predicting weekly averages of PM<sub>2.5</sub> and other derived quantities such as annual averages within three southeastern states. The PM<sub>2.5</sub> field is represented as the sum of semi-parametric spatial and temporal trends, with a random component that is spatially correlated, but not temporally. These authors apply a variant of the expectation-maximization (EM) algorithm to account for high percentages of missing data. Sahu and Mardia (2005) present a short-term forecasting analysis of PM<sub>2.5</sub> data in New York City during 2002 using a Bayesian KKF model.

Sahu *et al.* (2004) address a frequent criticism of spatial prediction using air pollution data from large-scale monitoring networks. Many of these networks were designed to capture peak pollution levels within urban, highly populated areas. For this situation, there is a potential for over-prediction within sparsely monitored rural areas with misleading prediction errors. Their paper presents a hierarchical space-time model that introduces two spatio-temporal processes, one capturing rural or background effects, the second adding extra variability for urban/suburban locations. They also consider the relationship of population density to fine particulate matter and incorporate non-stationary spatial and temporal covariance structure. Estimates of the probabilities of non-compliance with the proposed air quality standard for annual PM<sub>2.5</sub> are also provided, based on the weekly predictions of PM<sub>2.5</sub> for 2001. This spatial information is useful for determining where to place new sites to judge future compliance with air quality standards with measurements.

## 5.2 Climate and other environmental applications

A fast growing field is on fusing geophysical deterministic models with data. The field of combining observations with a deterministic model is known as data assimilation. It is critical in understanding and predicting geophysical systems such as in climate modeling and weather prediction. For example, Navier-Stoke equations are well known for describing turbulence but given a data how to input summary statistics such as principal components is one of the challenges. There have been emerging various Bayesian methods of data assimilation and "ensemble" filtering, see for example Bengtsson *et al.* (2003) Nychka (2000), Hoar *et al.* (2003), Kalnay (2003), Lorenc (2003) and Goldstein and Rougier (2005) The thesis advanced in the last article has a great deal of potential in many applications.

In a recent paper Glasbey and Allcroft (2005) develop a spatio-temporal auto-regressive moving average (STARMA) model for the solar radiation microclimate in Edinburgh. Knowledge of the statistical characteristics of solar radiation over space and time are needed for the design and evaluation of solar energy systems. Solar radiation was recorded at a pair of sites in Edinburgh every 30 seconds for two years, with the sites changed each month (Glasbey *et al.*, 2001). Although STARMA models can be computationally expensive, they show that by working on a torus in space, the order of dimension of calculations can be substantially reduced. They overcome the spatial sparsity of the data by assuming that space and time are interchangeable, and consider issues of model identification,

estimation and validation.

Spatio-temporal modeling of rainfall and precipitation is currently receiving a great deal of attention. Recent articles on rainfall modeling include Brown *et al.* (2001), Sansó and Guenni (1999, 2000) and Allcroft and Glasbey (2003). See Isham *et al.* (2005) for space-time simultaneous modeling of rainfall and soil mixture.

Sahu *et al.* (2005) reconstruct rainfall fields by combining two data sources: rainfall intensity as measured by ground raingauges (rain mm to the nearest 10th of one millimeter) and radar reflectivity. Radar raingauges are increasingly used to reconstruct rainfall fields since they are able to provide spatially continuous images of precipitation for short and regular time intervals; ground raingauges, on the other hand, provide more accurate and direct estimates of rainfall intensity. They compare two spatio-temporal models in order to enhance the predictions. Jona Lasinio *et al.* (2005) provide further examples on rainfall modeling using a Bayesian KKF model.

### 5.3 Social sciences

Sociological and socio-economic phenomena usually co-vary over time and space and the empirical study of them generally involves analysis of complex data sets obtained from complex sample surveys. Multilevel multiprocess models for survey data that include longitudinal, event history and/or spatial information have the potential to reveal greater insight into social processes than methods that model one process at a time or investigate the association structure across processes at a single time. Furthermore, they permit the investigation of individual changes over space and time rather than changes at an aggregate level.

Cohort or panel studies provide a rich source of longitudinal information. Investigation using models that allow temporal and spatial information from such studies with other covariates including geographical information can lead to very fruitful research. However, there are still concerns regarding the release of spatial information and confidentiality of the data providers.

There are research articles where confidentiality is not an issue or those have been dealt with using appropriate methods before modeling is considered. For example, Gelfand *et al.* (2004a) model house price data in the city of Baton Rouge, Louisiana, USA. They propose a rich class of spatio-temporal models under which each property is point referenced and its associated selling price modeled through a collection of temporally indexed spatial processes.

In a discussion paper Gelfand *et al.* (2004b) further develop methodologies using a computationally manageable class to build valid cross covariance functions referred to as the linear model of coregionalization (LMC). They also propose the notion of a spatially varying LMC (SVLMC) providing a very rich class of multivariate non-stationary processes with simple interpretation. They illustrate the use of the SVLMC with application to more than 600 commercial property transactions in three quite different real estate markets, Chicago, Dallas and San Diego.

Spatio-temporal models are also useful for analyzing spatial interaction of crime incidents, for example Kakamu *et al.* (2005) analyze the development of eighteen types of criminal records in Japan for the period 1990 to 2001 across 47 prefectures with spatial lag and spatio-temporal heteroscedasticity. They explore the hypothesis that crime data are related to socio-demographic variables in Japan.

## 5.4 Geostatistics and hydrology

Many concepts in spatial statistics originated from their use in geostatistics. See Diggle *et al.* (1998) for a recent review in model based geostatistics. Kent and Mardia (2002) provide a unified approach to spatio-temporal modeling through the use of drift and/or correlation in space and/or time to accommodate spatial continuity. For drift functions, they have emphasized the use of so called principal kriging functions, and for correlations they have discussed the use of a first order Markov structure in time combined with spatial blurring.

In a recent article Dryden *et al.* (2005) consider non-stationary spatio-temporal models in an investigation into karst water levels in western Hungary. A strong feature of the dataset is the extraction of large amounts of water from mines, which caused the water levels to reduce until about 1990 when the mining ceased, and then the levels increased quickly. They discuss some traditional hydro geological models which might be considered appropriate for this situation, and various alternative stochastic models. In particular, a separable space-time covariance model is proposed which is then deformed in time to account for the non-stationary nature of the lagged correlations between sites. Suitable covariance functions are investigated and then the models are fitted using weighted least squares and cross-validation. Forecasting and prediction is carried out using spatio-temporal kriging. They assess the performance of the method with one step ahead forecasting and make comparisons with naive estimators. Various other relevant references are given in this paper and also how it relates to the previous methodology.

## 5.5 System biology

The idea is to look at dynamics in a range of bioinformatics areas. In very broad terms, this would involve developing/extending and applying statistical/reverse-engineering tools to time series. Much of the data on protein structure and gene expression is limited to snapshots in time. However, since the data are typically very partial and noisy, their analysis is often limited in its success and in its power. Access to time series, where available, and extension of the analysis to dynamics should yield important insight. Possible applications areas include protein dynamics, gene-protein interaction, gene network dynamics, etc. There are other challenges in spatio-temporal modeling in this field, see for example Costas (2002).

”One of the greatest related challenges will be combining all genetic, molecular, geometrical and environmental effects into a model with a coherent set of spatiotemporal dynamics, and how to estimate experimentally the involved parameters..... While it is important to bear in mind that evolution will very likely not be explained as the development of optimal shapes for performing specific tasks implied by the environment, the shape-function paradigm will still essentially be involved with adaptation and fitness. In this sense, powerful approaches are required not only to characterise biological morphology, but also to estimate the degree of fitness of specific morphologies with respect to specific tasks. This important possibility involves the whole range of spatial and temporal scales, from proteins to environment, passing through cells, organs, members and individuals. Such endeavours will imply the application of concepts and tools from physics, image and shape analysis and dynamical systems.”

## 5.6 Archaeology

Spatio-temporal problems in archaeology and palaeo-environmental research (including climate change) cannot be tackled readily using standard models because of the presence of uncertainty on both the temporal and the spatial scales. Typically, the temporal information arises from chronometric dating methods, such as radiocarbon or uranium-series dating, which lead to estimated rather than exactly known calendar dates. Along side this, past landscapes were often very different from modern ones in important and poorly understood ways. This means that in order to reliably make inferences on either or both of the space and time scales, we need carefully devised models that take account of the uncertainty and provide probabilistic solutions to the questions posed.

Until recently, researchers working on such problems have taken one of two approaches. Some, like the famous work by Ammerman and Cavalli-Sforza (1973), use deterministic models to represent the spread of populations across landscapes without formally fitting them to real data. Others use stochastic models, but have not attempted to represent changes in space and time in the same model. Blackwell and Buck (2003), for example, illustrated the use of a fully Bayesian model in which the uncertainties arising from the radiocarbon dating evidence are formally accounted for, but the spatial information is not explicitly modeled at all.

## 6. Discussion

In this review we have discussed the recent developments in the models for spatio-temporal data. The associated issues of model fitting, choice, diagnostics, validation and prediction have not been touched upon at all, though they are very important. The Bayesian framework together with the Markov chain Monte Carlo methods are becoming increasingly popular, see Chapter 4 of Banerjee, Carlin and Gelfand (2004) for a broad overview.

We have not discussed many areas like brain imaging, modeling wildlife and tracking wild animals, tree defoliation in space and time, river flows, disease epidemic and many more. A book of abstract giving a recent snapshot of activities has been edited by Sahu (2005). There is also a growing literature on modeling mis-aligned data, see, for example the references in Banerjee, Carlin and Gelfand (2004, Chapter 6).

At present there is no general purpose software for model fitting and prediction, although the BUGS (Spiegelhalter *et al.*, 1996) software (**B**ayesian inference **U**sing **G**ibbs **S**ampling, available from `mrc-bsu.cam.ac.uk`) can handle some of the problems. Clearly more research on unification is needed before such a tool can be developed. The development we have presented in Section 4 is one such attempt as it presents three different modeling strategies unified through hierarchical models.

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