Some Problems on the Estimation of Marshall-Olkin Copula Parameters

Sulla stima dei parametri della copula di Marshall-Olkin

Silvia Angela Osmetti[1], Paola Maddalena Chiodini[2]
[1] Dipartimento di Scienze statistiche, Università del S.Cuore, Milano
[2] Dipartimento di Statistica, Università degli Studi di Milano-Bicocca
e-mail: silvia.osmetti@unicatt.it

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1. Introduction

This job discusses the problem of the study of the bivariate distributions used in reliability analysis by the meaning of a copula. The copula (Nelsen, 2006) is an helpful tool for handling multivariate distributions with given univariate marginals. For multivariate distribution, the univariate marginals and multivariate dependence structure can be separated, and the dependence structure can be represented by a copula. Copula allows to construct new multivariate distributions with arbitrary marginals.

In this paper we consider the survival copula by Marshall and Olkin. This copula became by Marshall-Olkin bivariate exponential distribution (Marshall, Olkin, 1967). This model has been proposed to study complex system in which the two components are not independent. We extend this model using the copula and different marginal distribution in order to construct bivariate survival functions.

The aim of this paper is to evaluate the problem of estimating the parameters of these distributions. We propose an easy procedure based on the method of moments, an alternative procedure to the maximum likelihood estimation generally used in literature. The moment's method is used to estimate the distribution parameters in two steps: in the first step we estimate only the parameters of marginal distributions and in the second step we estimate only the copula parameter.

The study of simulation is made either for the case of a complete or censored sample, this is done in order to evaluate the properties of the estimators for both cases.

2. The model and method of estimation

The Marshall-Olkin copula is a function C:[0,1]×[0,1]→[0,1]:

$$C(u,v) = u \cdot v \cdot \min(u^{-\theta}, v^{-\theta}) \tag{1}$$

With an appropriate extension of its domain to $\Re^2$, the copula is a joint distribution function with marginals uniform on [0,1]. This copula depends on a parameter $\theta\in[0,1]$ (we consider the case in which the variables are exchangeable) that reflexes the dependent structure existing between the marginals, from the stochastic independent situation (θ=0) to the situation of co-monotonicity (θ=1). Using exponential margins in the copula leads to the bivariate survival distribution of Marshall-Olkin, using Weibull allows us to obtain a bivariate Weibull distribution and through the margin distributions of extreme value we obtain a bivariate distribution of extreme value.

The likelihood method is usually used in literature to solve the issue of estimating the parameters of these survival functions. In this paper we use the method of moments, which is preferred to other estimation methods for its simpler mathematic form. This procedure is used to estimate both the copula parameter and the parameters of the marginal distributions considered. Parameters of the marginal distributions are estimated as a function of the sample moments of marginal variable. We estimate the θ copula parameter as a function of the sample mixed moment between $z_1=-ln(u)$ and $z_2=-ln(v)$:

$$\hat{\theta} = 2 - 2 \cdot (E(z_1 z_2))^{-1} \tag{2}$$

2.1. Simulation experiment

The estimate procedure described is verified thorough the use of several simulation experiments with Monte Carlo method, generating 2000 samples both in the case of complete and censure sampling. We assume a growing sample size *n*, from 100 to 1000, and several marginal distributions (exponential, Weibull, extreme value distribution) with different parameter values. The goodness of the obtained estimations is valued calculating the bias and the mean square error (MSE). To value the goodness of the estimation of the parameter of the copula we also compare the value of Kendall's τ estimated thorough θ parameter estimation and the real one. The efficiency of the estimation of the shape parameters of Weibull distribution is valued comparing their variance with the lower limit of the inequality of Rao-Cramér.

In the complete sampling we see a good stability of the estimations: the bias and the MSE are usually lower than the one tenth of the real parameter. Once increase in the sample dimension *n* we see an improving of the parameter estimates due to a strong decrease of the bias and the MSE, which are inversely proportional regarding *n*. The estimation of the shape parameters registered a satisfactory efficiency: the variance is close to the inferior limit of the inequality of Rao-Cramér.

We obtain a satisfying result also in the censured sampling (II type of censure). The censure is done after the observation of 80% of the system failures. In this case we observed a standard increase of the bias and the MSE compared to ones obtained with the complete sampling. However we obtained a considerable reduction of the length of the experiment and therefore a decrease of the cost of the experiment.

References

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