

A Well-being Index Based on the Weighted Product Method

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Abstract It has long been accepted that the GDP per capita cannot alone explain the well-being in a geographical area. Several have been the attempts to construct alternative, non-monetary, indices of well-being by aggregating a variety of individual indicators that represent different dimensions of well-being. The most famous, in Italy, are the quality of regional development index (QUARS) proposed by the campaign “Sbilanciamoci!” and the quality of life index published by “Il Sole 24 ore”. An issue often not solved, from a methodological point of view, concerns the comparability of the data over time. In this work, we propose a ‘static’ and a ‘dynamic’ well-being measure based on the application of the Jevons index to the socio-economic indicators. The obtained indices are closely related and allow synthetic spatial and temporal comparisons of the level of well-being.

Key words: composite index, well-being, Jevons index

1 Introduction

In the scientific context, many attempts of well-being indices have been presented by different Institutions (Universities, Statistics Offices, International Organizations). In Italy, there are two interesting indices: the quality of regional development index (QUARS) proposed by the campaign “Sbilanciamoci!” and the quality of life index published by “Il Sole 24ore”.

Recently, the Italian National Institute of Statistics has launched a series of studies in order to measure and evaluate the progress of Italian society. The aim of the project, called BES, is to construct a set of measures of the various dimensions of wellness at regional level and for particular categories of people (e.g., male and female).

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One of the main problems in order to construct composite indices is the choice of a method that allows time comparisons. As is known, there are several procedures for the normalization of the data, most of which use ‘relative’ parameters (e.g., the average value, the minimum or the maximum of a given year).

These parameters affect both the QUARS, which is based on z -scores, and the index proposed by “Il Sole 24ore”, that uses a function of ‘distance from the best performer’. In the case of the Human Development Index (HDI), the problem has been overcome by using a re-scaling of the indicators in the range (0, 1) with limits independent from the observed values in a given year. This solution may lead to future values outside the predetermined range and the only alternative is to recalculate the index values for the past years.

In this paper, we propose an application of the Jevons index to the indicators of well-being that allows to build, for each unit, both a ‘static’ index, for regional comparisons, and a ‘dynamic’ index for time comparisons, in a not full compensatory perspective. In Section 2, a description of the method is reported and in Section 3 an application to real data is proposed.

2 ‘Static’ and ‘dynamic’ well-being index

The weighted product method is one of the major techniques in composite index construction since it represents a trade-off solution between additive methods with full compensability and non-compensatory approaches [4]. When an unweighted geometric mean of ratios - such as the Jevons index - is computed, the obtained result satisfies many desirable properties from an axiomatic point of view [2].

Let x_{ij}^t the value of the indicator j for the region i at time t ($j=1, \dots, m; i=1, \dots, n; t=t_0, t_1$). A ‘static’ well-being index may be defined as follows:

$$SWI_i^t = \prod_{j=1}^m \left(\frac{x_{ij}^t}{x_{rj}^t} 100 \right)^{\frac{1}{m}}$$

where x_{rj}^t is the reference or base value, e.g., the national average. Therefore, values of SWI that are higher (lower) than 100 indicate regions with above (below) average performance.

In order to compare the data from time t_0 to t_1 , for each region, we can construct a ‘dynamic’ well-being index given by:

$$DWI_i^{t_1/t_0} = \prod_{j=1}^m \left(\frac{x_{ij}^{t_1}}{x_{ij}^{t_0}} 100 \right)^{\frac{1}{m}} .$$

For the ‘circularity’ or ‘transitivity’ property of the index number theory, SWI and DWI are linked by the relation:

$$DWI_i^{t_1/t_0} = (SWI_i^{t_1}/SWI_i^{t_0}) DWI_r^{t_1/t_0} .$$

Note that the ‘dynamic’ well-being index is similar to the Canadian Index of Well-being [3], except for the aggregation function. The Canadian approach is full compensatory since a simple arithmetic mean of ratios is used. We think that a multiplicative approach, such as in the new HDI, is preferable from both an axiomatic point of view (property of the index) and a conceptual point of view (full compensability is not realistic) [1].

3 An application to the Italian regions

In order to show the calculation of SWI and DWI, we consider a set of indicators of well-being in the Italian regions in 2005 and 2009.

The variables used are: Sporting activities, Close to supermarkets, Green space, Public transport, Parking provision, Children’s services, Elderly home care.

The data matrix is reported in Table 1.

Table 1: Individual indicators of well-being in the Italian regions (Years 2005, 2009)

Regions	2005							2009						
	Sporting activities	Close to supermarkets	Green space	Public transport	Parking provision	Children’s services	Elderly home care	Sporting activities	Close to supermarkets	Green space	Public transport	Parking provision	Children’s services	Elderly home care
Piemonte	34,1	60,3	42,0	189,8	12,5	28,6	1,8	34,1	69,0	42,5	199,3	17,1	37,1	2,3
Valle d’Aosta	33,9	52,7	23,2	544,0	5,3	100,0	0,1	46,3	58,6	26,2	580,0	8,4	78,4	0,4
Lombardia	37,7	69,9	27,6	230,1	20,0	54,6	3,2	36,5	68,9	28,6	227,7	24,1	62,5	4,1
Trentino-Alto Adige	53,1	72,2	71,2	190,7	28,6	75,8	0,6	48,2	71,9	70,3	192,9	34,5	83,8	0,8
Veneto	39,4	65,8	58,7	122,5	39,8	42,7	5,0	39,6	70,1	62,3	124,4	42,2	70,2	4,8
Friuli-Venezia Giulia	36,7	72,7	21,8	257,4	11,9	53,0	7,9	37,5	74,6	22,1	258,1	12,0	83,6	7,7
Liguria	26,6	67,9	35,3	312,5	23,1	75,3	3,1	27,6	70,6	35,4	311,0	22,3	64,3	3,4
Emilia-Romagna	32,4	71,1	158,5	81,0	24,4	78,0	5,4	36,8	69,3	157,7	83,0	24,0	88,0	8,3
Toscana	30,4	68,7	152,5	106,0	18,6	78,0	2,1	33,1	64,3	152,1	108,4	20,9	74,6	2,2
Umbria	31,2	65,9	192,1	162,4	27,4	51,1	4,1	32,3	73,7	187,6	162,8	26,9	63,0	7,6
Marche	31,4	76,0	185,8	157,2	9,2	45,9	3,3	32,2	67,4	186,1	157,7	15,3	55,7	3,6
Lazio	33,7	74,3	127,4	124,5	6,5	30,4	3,3	29,4	74,7	121,0	132,3	7,0	30,7	4,0
Abruzzo	28,9	55,5	714,5	93,5	5,3	26,2	1,8	31,0	63,0	710,0	93,5	21,1	52,1	4,8
Molise	23,2	52,1	18,3	177,2	1,3	2,9	6,1	22,0	58,7	18,5	177,2	1,2	7,4	2,4
Campania	22,3	59,3	24,8	227,3	7,3	39,2	1,4	21,1	60,0	25,9	218,0	5,9	50,5	1,9
Puglia	25,8	70,3	7,8	114,3	7,3	27,5	2,0	23,8	69,6	8,1	122,0	8,2	44,2	2,0
Basilicata	24,4	55,5	547,9	84,9	2,4	32,8	3,9	27,1	65,2	545,6	87,4	2,3	21,4	5,1
Calabria	24,5	55,1	19,7	159,6	20,3	7,8	1,6	24,8	56,4	20,8	172,8	19,5	15,6	2,5
Sicilia	21,5	63,6	71,5	72,2	3,4	33,3	0,8	22,5	68,6	73,3	75,7	6,5	34,6	1,1
Sardegna	31,1	75,9	86,4	55,7	16,8	17,2	1,1	28,2	78,3	85,9	56,6	16,9	20,4	2,3
Italia	31,3	67,1	93,5	118,8	14,4	42,8	2,9	31,1	68,5	93,6	122,1	16,2	51,7	3,6

Source: <http://www3.istat.it/ambiente/contexto/infoterr/assi/asseV.xls>

Table 2 presents the results. Note that the base value of the ‘static’ indices (SWI_{05} and SWI_{09}), for each region, is the national average (Italy), while the base of the ‘dynamic’ index ($DWI_{09/05}$) is the value for the year 2005.

As we can see from the table, not necessarily each relative increase corresponds to an absolute one and vice versa. For example, from 2005 to 2009, Toscana shows a reduction of the level of well-being compared to the national average ($SWI_{05}=113.4$ vs. $SWI_{09}=107.3$), though the values of the individual indicators, on the whole, are

increased ($DWI_{09/05}=102.6$). This is due to a greater rise of the performances of the other regions which has produced a large increase of the national average in 2009.

Table 2: Static and Dynamic Well-being Index (Years 2005, 2009)

Region	SWI05	SWI09	DWI09/05	Region	SWI05	SWI09	DWI09/05
Piemonte	82,1	87,5	115,6	Marche	113,1	115,0	110,3
Valle d'Aosta	63,8	75,5	128,4	Lazio	93,4	87,8	102,0
Lombardia	105,0	104,5	107,9	Abruzzo	93,5	137,3	159,3
Trentino-Alto Adige	106,9	105,1	106,7	Molise	41,6	38,5	100,5
Veneto	120,9	122,5	110,0	Campania	68,4	65,9	104,5
Friuli-Venezia Giulia	108,6	107,5	107,4	Puglia	55,3	55,6	109,1
Liguria	114,7	105,3	99,6	Basilicata	89,7	83,7	101,3
Emilia-Romagna	132,7	134,3	109,9	Calabria	59,5	65,7	119,8
Toscana	113,4	107,3	102,6	Sicilia	54,9	60,0	118,5
Umbria	136,6	143,6	114,1	Sardegna	70,4	73,6	113,5

4 Conclusions

The comparability of the data over time is a central issue in composite indicators construction. Normalization methods such as ranking and standardization (z -scores) allow relative comparisons only. Re-scaling in range (0, 1) and indicization by 'distance to a reference' measures allow to evaluate absolute changes when the limits or the reference value are independent from the observed data. Another factor that may affect the comparability is the aggregation method, e.g., when the weights are based on Principal Components Analysis or Factor Analysis.

In this paper we propose a method based on the indices number properties for constructing two consistent indices of well-being: a 'static' index for spatial comparisons and a 'dynamic' index for temporal comparisons.

The method is based on a multiplicative approach and may be applied to different domains without loss of comparability. For example, it is possible to compute the indices for gender and compare the values obtained with others domains.

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