Assessing Multivariate Measurement Systems in Multisite Testing

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Abstract Multisite testing system refers to the simultaneous testing of products using multiple measurement instruments in parallel. It is important to perform measurement system analysis (MSA) on a multisite testing system in order to assess its testing capability. Multivariate measurement systems are by now so common that studies concerning MSA cannot be restricted to the univariate framework. In this work, we discuss the use of a statistical test based on the largest eigenvalue of a Wishart distribution for performing multivariate MSA in a multisite testing framework.

Key words: Multivariate measurement system analysis, Wishart distribution, Eigenvalues

1 Introduction

A multisite testing system [1] has multiple test instruments in parallel where each test instrument independently measures multivariate quality characteristics. An important assumption in multisite testing systems is that all instruments are expected to have the same level of measurement accuracy and precision after the initial instrument set up. However, uncertainty in measurements can be introduced by many factors such as the differences in instrument operators and wear-out development. From the above considerations it follows the importance of performing measurement system analysis (MSA) on a multisite testing system to help ensure the validity of the resultant data. In practice, it is difficult to distinguish the faulty

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measurements caused by a defective measurement device from those measurement variations caused by the normal differences across operators, products, and instruments. [1] proposed an online statistical process control multivariate MSA approach using principal component analysis (PCA) to detect faulty test instruments in a multisite testing system. The method proposed by [1] is interesting and effective, however it appears complicated since it requires several steps: to map the measurement data collected from each instrument into the orthogonal space of cross-instrument principal components and into the orthogonal space of distinct-instrument principal components; to compute the differences between the corresponding principal component values; to setup separate control charts for the most important principal components. In this framework [1] considered a faulty instrument as one whose principal component values were beyond the three sigma control limits of the principal component values of all instruments. In this work, we propose a statistical test which can be useful for assessing multivariate measurement systems in a multisite testing context. The aim is to provide a less complicated statistical method for detecting one or more faulty instruments.

2 Multivariate MSA in Multisite Testing

A multivariate multisite measurement system has m parallel measurement instruments, each instrument measures p quality characteristics. Let x_{ijk} denote the kth $(k = 1,2,...,p)$ quality characteristic, measured by the ith $(i = 1,2,...,m)$ instrument for the jth $(j = 1,2,...,n)$ product or part. Let $\mathbf{X} = [X_1, X_2, ..., X_m]'$ be a $(m \cdot n) \times p$ matrix of measurements, where X_i is a $n \times p$ matrix of the values of the p quality characteristics measured on n parts by the instrument i. We assume that $X_i \sim N_p(\mu_i, \Sigma_i)$ and $\mathbf{X} \sim N_p(\mu, \Sigma)$. In the multisite testing framework Σ is the cross-instruments covariance matrix and Σ_i is the covariance matrix of the instrument i. If we assume that the m instruments have the same level of precision and accuracy, then they belong to the same population and their mean vector and covariance matrices are the same: $\mu_1 = \mu_2, ..., = \mu_m = \mu$ and $\Sigma_1 = \Sigma_2, ..., = \Sigma_m = \Sigma$. Without loss of generality let we assume calibrated measurement instruments ($\mu_1 = \mu_2, ..., = \mu_m = \mu$), thus we focus only on the measurements variability. An instrument becomes faulty if $\Sigma_i = \Sigma + \Sigma_{e_i}$ with Σ_{e_i} positive definite. In single site testing systems multivariate MSA methodologies are used to evaluate the components of variations in the precision of the measuring devices. [2] proposed multivariate assessments criteria based on a MANOVA approach. However, as noted by [1], these measures may experience several difficulties in detecting a faulty measurement instrument in a multisite measurement system, where the variations among the parallel instruments adds another layer of complexity.

3 The proposed method

If all the m instruments have the same precision, then the hypothesis $H_0 : \Sigma_i = \Sigma$ for $i = 1,2,..,m$ holds. If one ore more instruments become faulty the appropriate hypothesis is $H_1 : \Sigma_i = \Sigma + \Sigma_{e_i}$ for at least one i. Let us define the matrix $\Delta = \Sigma_i \Sigma^{-1}$ and let $\lambda_{\Delta 1}$ the largest eigenvalue of the matrix Δ. According to the present notation, let $x_{ij.}$ denote the jth row $(1 \times p)$ of matrix X_i. Thus, $\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij.}$, is the sample mean of the measurements from the instruments i and $S_i = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ij.} - \bar{x}_i)'(x_{ij.} - \bar{x}_i)$ is the unbiased estimator of Σ_i. If the hypothesis $H_0 : \Sigma_i = \Sigma$ holds, then $(n-1)S_i\Sigma^{-1} \sim W(I, n-1)$ and a test for H_0 versus H_1 can be performed using the statistic $(n-1)\widehat{\lambda}_1$, where $\widehat{\lambda}_1$ the largest eigenvalue of the matrix $S_i\Sigma^{-1}$. The hypothesis H_0 is not rejected if $\widehat{\lambda}_1 < u_\alpha$, where u_α is the upper α percentage point of the largest characteristic root of a Wishart matrix. In order to investigate the usefulness of the proposed test we design a simulated experiment by adapting a real case examined by [3]. The data concern a three-dimensional process with cross instrument covariance matrix assumed known

$$\Sigma = \begin{bmatrix} 0.0021 & 0.0008 & 0.0007 \\ 0.0008 & 0.0017 & 0.0012 \\ 0.0007 & 0.0012 & 0.0020 \end{bmatrix} \tag{1}$$

and $\mu = [2.2, 304.8, 304.8]$. In our experiment we consider a multisite system with $m = 3$ parallel measurement instruments, where instrument $i = 1$ has $\Sigma_1 = \Sigma$ and instrument $i = 2$ has $\Sigma_2 = \Sigma + \Sigma_{e_2}$ with

$$\Sigma_{e_2} = \begin{bmatrix} 0.0003000 & 0.0000882 & 0.0001103 \\ 0.0000882 & 0.0002000 & 0.0000794 \\ 0.0001103 & 0.0000794 & 0.0002300 \end{bmatrix} \tag{2}$$

As far as instrument $i = 3$ is concerned, in order to cover a range of different operative situations, we examine several scenarios. In detail, we examine a situation where initially the measurement instrument covariance matrix is $\Sigma_{e_3} = \Sigma_{e_2}$, then the diagonal elements of Σ_{e_3} increase from their initial values, say $\sigma^2_{e_{3kk}}$, to $\delta\sigma^2_{e_{3kk}}$, with $\delta = 3$ (step of 0.1), while the measurement error correlation structures remain unchanged. We used the `R` software to generate, for each value of δ, a sample of size $n = 51$ from $X_i \sim N_p(\mu, \Sigma_i)$ for $i = 1,2,3$. Furthermore, we used the `R`-package **RMTstat** [4] for computing the the critical value of the test, which for $p = 3$, $n = 51$ and $\alpha = 0.05$ is $u_\alpha = 81.998$. Table 1 shows the values of the statistic $(n-1)\widehat{\lambda}_1$ computed from the simulated samples for instruments 1, 2 and 3.

Examining the results reported in Table 1 we can see that for instrument $i = 1$ the hypothesis is H_0 is correctly never rejected, since in the experiment we have assumed an adequate measurement instrument: $\Sigma_1 = \Sigma$. For $i = 2$ we have simulated an instrument with covariance matrix Σ_2 slightly increased respect to the cross-instruments covariance matrix Σ. This intermediate stage of a minor worsening of the measurement capability is captured by the proposed test: in three cases H_0 is

Table 1 Values of $(n-1)\widehat{\lambda}_1$ computed from the simulated experiment

δ	Instrument 1	Instrument 2	Instrument 3
1.0	79.111	56.442	73.138
1.1	63.606	68.591	66.693
1.2	54.313	69.657	85.857
1.3	80.325	74.069	69.702
1.4	71.605	56.699	63.700
1.5	75.005	75.459	88.608
1.6	61.425	70.726	63.759
1.7	59.022	79.967	73.250
1.8	77.451	71.100	85.714
1.9	70.394	83.366	69.249
2.0	73.467	76.080	92.879
2.1	68.183	67.741	73.944
2.2	50.038	51.068	109.293
2.3	54.724	71.241	90.925
2.4	69.494	79.939	92.893
2.5	64.326	94.472	77.357
2.6	53.985	76.875	100.108
2.7	69.197	65.413	89.178
2.8	64.703	60.796	86.983
2.9	50.631	61.752	89.096
3.0	65.332	89.184	84.368

rejected. Finally, let us examine the very different case of instrument $i = 3$ where the experiments involve covariance matrices Σ_{e_3} with increasing diagonal elements as functions of δ. In this cases H_0 is often rejected in particular when δ increases. Tentative conclusions based on previous results have to be taken with some caution, since we are considering a limited case study. However, keeping this caution in mind, we can conclude that the proposed test may be useful for performing multivariate MSA in multisite testing systems since it appear able to detect decreasing performances in measurement instruments.

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