Indexing the Worthiness of Social Agents

Giulio D'Epifanio

Abstract

In evaluating the performance of social agents, a method is considered which, by interpreting the "worthiness" in a probabilistic setup, quantifies the ordinal levels of the outcome scale. A worthiness-based index is proposed.

Key words: indexing, evaluation, ordinal scale quantifying, worthiness

1 Introduction

The reference problem is to analyze the data in table (1a), in order to compare the performance of social agents (e.g. providers of social services, as hospitals, schools, etc.). The performance of any agent u may be associated to the "social behavior" described by the distribution $p[u] := (p_0, p_1, \ldots, p_L)[u]$ realized, on the set of the individuals which it governs, upon the $(L+1)$ levels of an ordinal classifier of outcome Y. The policy maker (PM) wants that the agents assessments are standardized on the "reference behavior", which is associated to a certain agent A_0, described by data[1] in table (1b).

University of Study of Perugia, Department of Economic, Finance and Statistics, Italy
ggiulio@unipg.it

[1] These "reference data" have been assigned, perhaps heuristically (e.g. choosing a concrete "best practice" or a virtual "behavior template for reference"), through some "a priori" criterion specified by the PM in the evaluation design.

level of performance Y:	I	II	III	IV	V	VI	VII
actual agents:							
agent A1	0	16	24	31	12	0	4
agent A2	4	28	65	107	26	1	3
agent A3	4	42	71	102	33	0	2
agent A4	4	71	112	194	53	0	4

(a) Data of the actual agents

Y:	I	II	III	IV	V	VI	VII
reference agent A0	21	115	272	370	124	5	13

(b) Data of the reference agent

Table 1

The abstract theory of utility (e.g. see [1]) provides evaluation indexes, which are consistent on general principles of rationality in choices. But, they demand that the ordinal levels of outcome scale Y are to be quantified. Thus, the practical methodological question arises in assigning values to the ordinal levels of outcome Y, which will enter the evaluation index, in such a way that the instanced index is "pertinent to the purpose" of the PM, fully standardized on the assumed evaluation design. Although commonly used, the "Likert-like" practice, of setting the distance between adjacent levels as constant, seems excessively "naive" to be justified. However, even the more sophisticated statistical approaches in ordinal data analysis (e.g. see [4]) may be lacking of any "pertinence to the purpose[2]" of the PM. On the other hand, merely subjective approaches may be difficult to justify institutional benchmarking. In this communication, an indirect approach (e.g. see [3]) is delineated to quantify the ordinal levels of the outcome Y. It adopts a criterion, which evaluates achieving of scheduled goals, based on the concept of worthiness. The "worthiness increases" are interpreted on a probabilistic setup. Then, a practical method is outlined for eliciting the "worthiness increases", between adjacent levels of the scale, which are standardized on the behavioral data of the reference agent.

2 Indexing

Recode, nominally, the ordinal levels of the outcome Y so that $Y \in \{0, 1, \ldots, L\}$. Suppose now that, in its evaluation design, the PM

- schedules, ideally, the following chain of (Guttman like ordered) goals

$$O_0 \preceq O_1 \preceq O_2 \preceq ... \preceq O_l \preceq ... \preceq \ldots \preceq O_{L-1} \preceq O_L := O_{Full}, \qquad (1)$$

  where, $O_l := (Y \geq l)$, $l := 0, \ldots, L$ is the l-th scheduled goal

[2] In effect, adhering to some technical criterion of "optimality in data-analysis" is not "*per se*" sufficient to provide evaluations which are also pertinent to the purpose of the PM.

- assigns the "increases of value[3]" $\omega_l := \Delta_{l-1}Val := Val(O_l) - Val(O_{l-1}) \geq 0$, $l := 1, \ldots, L$, where $Val(.)$ is a (non-negative, not decreasing in value through the levels $l := 0, \ldots, L$) value-function which is initialized to $Val(O_0) = 0$.

In this setup, the performance of any agent $u \in \{A_1, A_2, \ldots, A_p\}$ is described by the discrete distribution $(s(0;\omega), p_0(u); s(1;\omega), p_1(u); \ \ldots \ ; s(L;\omega), p_L(u))$. Here, $s(.;\ \omega)$ denotes any quantification function of the nominal levels $0, 1, \ldots, L$ of Y such that (up to rescaling and translations):$s(0;\ \omega) := Val(O_0) = 0 \leq \ s(1;\ \omega) := Val(O_1) = \omega_1 \leq \ s(2;\ \omega) := Val(O_2) = \omega_1 + \omega_2 \leq \ \ldots \ \leq \ s(L;\ \omega) := Val(O_L) = \omega_1 + \omega_2 + \cdots + \omega_L$. Instead, $p[u] := (p_0, p_1, \ldots, p_L)[u]$ is the distribution, actually realized by the governed individuals of agent u, on the ordinal levels of Y. Then, the theory of utility provides, as a particular instance (e.g. see [1], pp. 559) of more general functionals, the expectation-based index

$$u \ :\longmapsto W[p[u];\ Val(.)] := \sum_{l:=1}^{L} \omega_l \cdot (1 - F_Y[p[u]](l)) = \sum_{l:=1}^{L} s(l;\ \omega) \cdot p_l[u] \quad (2)$$

up to parameters $\omega = (\omega_1, .., \omega_L)$ of the quantification function $s(.;\ \omega)$ which should be specified by the evaluation design. Here, $F_Y[p]$ denotes the cumulative distribution such that $F_Y[p](l) = p_0 + p_1 + \cdots + p_{l-1}$.

3 Worthiness based indexing

Recalling (see [2]) the criterion of intrinsic worthiness[4], the "increases of worthiness"$\omega_l := \Delta_{l-1}Val(.)$ may be interpreted as follows.

> Let $\mathcal{P}^*$denote the population of the (real or perhaps virtual) individuals which are governed by the "reference agent" A_0. For any actual individual i, having achieved goal O_{l-1} on chain of goals (1), the higher "the risk of failing the next goal O_l", referring such risk on the population $\mathcal{P}^*$, the greater the "increase of worthiness" in performance which i gains "as if" it was in $\mathcal{P}^*$, whenever it actually achieves goal O_l.

Then, index (2) may be specialized to

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[3] Here, $\omega_l := \Delta_{l-1}Val$ is interpreted as the "standardized value increase" which is gained by any social agent in improving the condition of a "*standard individual*" from the current $(l-1)$th level of goal achievement to the next lth on chain (1).

[4] Consider hierarchical chain of goals *(1)*. Given that a certain goal O_{l-1} has been achieved, the greater the resistance, with reference to the evaluation framework, to also achieve the next pursued goal O_l, by continuing to improve, the greater the increment of value due to the intrinsic worthiness of who, effectively, is able to achieve it.

$$u \ :\longmapsto W[p[u];\, \omega^*] := \sum_{l:=1}^{L} \varphi_l(\frac{Pr\{Y = l-1; \mathcal{P}^*\}}{Pr\{Y \geq l-1; \mathcal{P}^*\}}) \cdot (1 - F_Y[p[u]](l)) \quad (3)$$

by setting the "value increases" $\omega_l := \Delta_{l-1} Val := Val(O_l) - Val(O_{l-1}) \geq 0$ to

$$\omega_l^* = \varphi_l(\frac{Pr\{Y = l-1; \mathcal{P}^*\}}{Pr\{Y \geq l-1; \mathcal{P}^*\}}) = \varphi_l(\frac{p_{l-1}}{p_{l-1} + p_l + \cdots + p_L}), \ \ l := 1, .., L \quad (4)$$

Continuous monotone functions $\varphi_l(.)$ of the rates may be chosen ([2]) for specifying some characteristics (e.g. the additivity) of the scale.

3.1 Model based indexing

A criterion might be adopted by the PM which want justify behavior differences, in the "worthiness increases", on the basis of differences of condition. Thus, the increases of worthiness (4) might be modeled by using some probabilistic model, through *"reference domains*" $x := X \in \{x_1, \ldots, x_R\}$. A global evaluation index, for instance based on a sequence of logistic models, may be the following:

$$u \longmapsto \sum_{r:=1}^{R} q_r \cdot \{ \sum_{l:=1}^{L} \varphi_l(\frac{\exp(\widehat{a}_l + \hat{b}_l x_r)}{1 + \exp(\hat{a}_l + \hat{b}_l x_r)}) \cdot (1 - F_{Y|x_r}[p[u]](l)) \} \quad (5)$$

where parameters $\hat{a}$ and $\widehat{b}$ were determined on reference population $\mathcal{P}^*$. Here, $q_r \geq 0$ ($\sum_{i:=1}^{R} q_r = 1$) weights the reference domain for the status x_r. The weights should represent the political relevancy of the "social reference domains" to the overall purpose of the PM.

References

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