A Multivariate VEC-BEKK Model for Portfolio Selection

Andrea Pierini and Alessia Naccarato

Abstract The use of bivariate vector error correction (VEC) models and Baba-Engl-Kraft-Kroner (BEKK) models is proposed for the selection of a stock portfolio (Markowitz portfolio) based on estimates of average returns on shares and the volatility of share prices. The model put forward is applied to a series of data regarding the prices of 150 shares traded on the Italian stock market (BIT) between 1 January 1975 and 31 August 2011.

Key words: Markowitz portfolio, vector error correction (VEC) model, BEKK model, cointegration

1 Introduction

The selection of a stock portfolio is broadly discussed in the literature, generally with reference to heteroskedastic regression models [1]. The models used in the case of multiple time series are of the vector autoregressive (VAR) type [3].

This paper proposes the use of vector error correction (VEC) and BEKK models for the selection of a stock portfolio. In other words, it addresses the problem of estimating average returns and the associated risk on the basis of the prices of a certain number of shares over time. This estimate is then used to identify the assets offering the best performance and hence constituting the best investments. While Campbell [3] proposes the use of a VAR(1) model, it is suggested here that use should be made of VEC models, which make it possible to take into account any

Andrea Pierini
University of Roma Tre, Department of Economics, Via S. D'Amico 77,00145 Roma, e-mail: apierini@uniroma3.it and Alessia Naccarato
University of Roma Tre, Department of Economics, Via S. D'Amico 77,00145 Roma, e-mail: naccarat@uniroma3.it

cointegration between the series employed and the market trend as measured by means of the Thomson Reuters Datastream Global Equity Italy Index [4].

Moreover, while Bollerslev, Engle and Wooldridge [2] employ diagonal vectorization (DVEC) models to estimate share volatility, the use of a BEKK model, as proposed here, makes it possible to extend the estimation procedure based on DVEC models so as to take into account also the correlation between the volatility of the series and the volatility of the market trend.

The series considered regard the Italian stock market (BIT), and specifically the monthly figures for the top 150 shares in terms of capitalization, from 1 January 1975 to 31 August 2011. The estimation procedure proposed for portfolio selection involves two phases.

In the first, a two-dimensional VEC model is developed for all of the 150 shares considered in order to obtain an estimate of the average stock market return. A BEKK model is then applied to the series of residuals thus obtained in order to estimate the volatility of the series.

The second regards the selection of shares for inclusion in the portfolio. Only those identified as presenting positive average returns during the first phase are considered eligible. For the purpose of selecting the most suitable of these, a new endogenous variable is constructed as the product of two further elements, namely the price-to-earnings ratio (P/E) and earnings per share (EPS). This variable, which indicates the intrinsic value of the share in question, is not constructed for the entire set of 150 shares but only for those presenting positive average returns in the first phase, as it would be pointless in the case of negative returns. The VEC-BEKK model is applied once again to this new series in order to estimate the intrinsic value of the shares, and the top 10 are selected for inclusion in the portfolio on the basis of the difference between this intrinsic value and the price estimated in the first phase.

A quadratic programming model is then employed to determine the quantities to be bought of each of the 10 shares selected. It should be noted that the variable P/E · EPS is estimated for each industrial sector.

2 Model Summary

A concise outline is now given of the phases involved in the selection of shares for inclusion in the portfolio as well as the quantity of shares to be bought for each type selected. The starting point is the $K = 150$ series, regarding the average returns $R_{k,t}$ on the shares, and the average return of the market $R_{M,t}$, $t = t_k, \dots, T, k = 1, \dots, K$. It should be noted in this connection that the length of the series considered is not homogeneous because not all of the joint-stock companies are quoted as from the same point in time. This aspect involves further complications in the estimation procedure.

Phase one :For each series, the model $VAR_2(p)$ is constructed for the random vector $y_t = [y_{1,t}, y_{2,t}]' = [R_{k,t}, R_{M,t}]'$

$$y_t = \mu_t + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_p + u_t \tag{1}$$

with $\mu_t = \mu_0 + \mu_1 t$, A_i matrix 2×2, $i = 1, \ldots, p$ of the unknown coefficents, and $u_t = [u_{1,t}, u_{2,t}]'$ the vector of errors such that $u_t \sim N(0, \Sigma_u)$.

Model (1) can be rewritten as follows to take into account and possible cointegration of the variables considered:
This model can be rewritten in the form of $VEC_2(p-1)$, which shows manifestly the possible cointegration, which we use in presence of it

$$\Delta y_t = \mu_t + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t \tag{2}$$

The AIC criterion is used to estimate the lag $\hat{p}$, with reference to model (2), and the LR test is carried out to ascertain the presence of cointegration.

Finally, the method proposed by Johansen [6] is applied to obtain the maximum-likelihood estimation (MLE) of the parameters $\mu_0, \mu_1, \Pi, \Gamma_1, \ldots, \Gamma_{p-1}$.

The Portmanteau test is used to ascertain the presence of correlation of residuals, the generalized Lomnicki-Jarque-Bera test for the normality of residuals, and the ARCH test to determine heteroskedasticity.

In the event of the latter test revealing the presence of heteroskedasticity, the BEKK(1,1) model is used to estimate the conditional variance-covariance matrix $\Sigma_t = cov(u_t|past) = ((\sigma_{i,j}(t))_{i,j=1,\ldots,n}$, which has the following structure:

$$\begin{aligned}
\begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{22,t} \end{bmatrix} &= \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ 0 & a_{22} \end{bmatrix} + \\
&+ \begin{bmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 & u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}u_{1,t-1} & u_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11,1} & a_{21,1} \\ a_{12,1} & a_{22,1} \end{bmatrix} + \\
&+ \begin{bmatrix} b_{11,1} & b_{12,1} \\ b_{21,1} & b_{22,1} \end{bmatrix} \begin{bmatrix} \sigma_{11,t-1} & \sigma_{12,t-1} \\ \sigma_{21,t-1} & \sigma_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11,1} & b_{21,1} \\ b_{12,1} & b_{22,1} \end{bmatrix}
\end{aligned} \tag{3}$$

Phase two: The estimates obtained in phase one are used to select the shares for which positive average returns are predicted. For the shares thus selected and for each industrial sector (IS), the model $VEC_2(p-1) - BEKK(1,1)$ is estimated for the random vector $y_t = [y_{1,t}, y_{2,t}]' = [((P/E) \cdot (EPS))_{h,t}, ((P/E) \cdot (EPS))_{IS_h,t}]'$, where $h = 1, \ldots, H$ is the index that identifies only the series with positive returns selected out of the initial 150.

On the basis of the $((P/E) \cdot (EPS))_{IS_h,T+1}$ and $R_{h,T+1}$ forecasts obtained in phase two, the shares are listed for each industrial sector in decreasing order with respect to the values of the difference between intrinsic value and expected price. The first $n = 10$ shares are thus selected to make up the portfolio.

Finally, in order to determine the quantities to be bought of each of the 10 shares selected, it is necessary to solve the Markowitz problem [7] by estimating the matrix of share volatility. To this end, let be $\hat{V}_t$ the estimator of the matrix $n \times n$ of volatility V_t for $t = T+1$, the elements of which are $v_{i,j}(t) = cov(R_t|past), i, j = 1, \ldots, n$. The elements of $\hat{V}_t$ are given by:

$$\hat{v}_{i,j}(T+1) = \begin{cases} \hat{\sigma}^{(i)}_{11,T+1|T} & \text{se } i = j \\ \hat{c}_{ij} & \text{se } i \neq j \end{cases} \quad (4)$$

with $\hat{C} = (\hat{c}_{i,j})_{i,j=1,...,n} = \sum_{t=t_{max}}^{T}(R_{i,t} - \overline{R}_i)(R_{j,t} - \overline{R}_j)'/(T - t_{max})$, $t_{max} = max\{t_i, t_j\}$, $i, j = 1, ..., n$, $n = 10$. On the basis of (4), the solution of the quadratic Markowitz problem $\min_{\{\omega:\omega'\mathbf{1}=\mathbf{1}\}} \omega' V \omega$ for the future time $T+1$ can be obtained as

$$\hat{\omega}_{opt,T+1} = \hat{V}^{-1}_{T+1}\mathbf{1}/(\mathbf{1}'\hat{V}^{-1}_{T+1}\mathbf{1}) \quad (5)$$

3 Results

Application of the model proposed in this work to the monthly figures for the 150 BIT shares with the highest level of capitalization indicates the following results:

a) an *optimal lag p* of 2 - 9 months.

In particular, the optimal lag is 2 months for 76% of the entire set of 150 shares. This means that just 2 months of observation are sufficient to predict the average returns on the vast majority of the shares considered.

b) The degree of *cointegration* proves equal to 2 for 80% of the 150 shares, 1 for 18% and 0 for the remaining 2%.

c) The *coefficients* of the models estimated in both steps of the procedure prove significant for almost all of the series considered.

d) The BEKK estimate of *volatility* for each share is between 0.001 and 0.01 for 87% of the series and never above 0.031

e) The *confidence interval* at the level of significance of 95% contains the actual value$_{T+1}$ in 89% of the series. The VEC-BEKK model can therefore be considered reliable for most of the series for the purposes of prediction.

f) The portfolio has a *monthly average return* of 1.2%, a *monthly standard deviation* of 0.578, and a *Sharpe index* of 0.021.

References

1. Bollerslev T., Engle R.F., Nelson D.B., 1994, "ARCH Model", Handbook of Econometrics, IV, 2959-3038, Elsevier Science, Amsterdam
2. Bollerslev T., Engle R.F., Wooldridge J.M., 1988, "A Capital Asset Pricing Model with Time-Varying Covariance", Journal of Political Economy, 96, 116-131
3. Campbell, J.Y., Chan Y.L., Viceira L.M., 2003, "A Multivariate Model of Strategic Asset Allocation" Journal of Financial Economics, 67, 41-80
4. Datastream Global Equity Indices 2008, user guide, issue 5,Thomson Reuters Ltd
5. Engle R.F. and Kroner, K.F., 1995, "Multivariate Simultaneous Generalized ARCH", Econometric Theory, 11, 122-150
6. Johansen S., 1995, "Likelihood-Based Inference in Cointegrated Vector Autoregressive models", Oxford University Press, Oxford
7. Markowitz H., 1952, "Portfolio Selection", Journal of Finance 7, 77-91