A Simple Risk-Adjusted CUSUM chart for monitoring binary health data

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Abstract When data consist of patients with a specific risk to develop an health event and we are interested in monitoring the incidence rate of this event, the standard Bernoulli CUSUM fails and a risk-adjustment needs. The risk-adjusted CUSUM chart proposed by Steiner et al. (2000) shows some limitations. To overcome these limits a simple risk-adjusted CUSUM is proposed and it may be considered a general, simple and efficient solution. Simulated and actual data sets illustrate the performance and the usefulness of the proposed procedure. An application to mortality data is also shown.

Key words: health data, CUSUM chart, surveillance, risk-adjustment, Bernoulli variate

1 Introduction

When health data consist in a continuous stream of binary individual observations (patients) Reynolds and Stoumbos (1999) developed the Standard Bernoulli cumulative sum (CUSUM) chart, designed to detect a shift in the overall in-control incidence rate p_0. However patients could present very heterogeneous characteristics in terms of risk factors and then of risk of event. Therefore, an adjustment for time varying in-control risks of event is required because the system could signal an alarm even though a shift in incidence rate has not occurred, but the baseline risk of event has modified due to changes in the patient mix and so in the underlying predisposed risk distribution.

For this purpose Steiner et al. (2000) proposed a Risk-Adjusted CUSUM chart. This method has been used particularly for monitoring surgical outcomes, but setting

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appropriate thresholds is not straightforward and there is no simple formula for constructing the boundary [4,1]. Furthermore, it remains sensitive in changes in the underlying predisposed risk distribution [2].

To overcome these problems we propose a Simple Risk-Adjusted CUSUM chart, very easy to implement and less sensitive to changes in the underlying predisposed risk distribution.

2 Materials and methods

In this section we briefly introduce the Standard Bernoulli CUSUM and the proposed Simple Risk-Adjusted CUSUM chart.

Standard Bernoulli CUSUM

Let observations y_t be distributed as a Bernoulli variate with parameter p, which is the probability that an event will occur. We assume $p = p_0$ when the process is in-control and we are interested in detecting a shift from p_0 to p_1 as quickly as possible. If this happens the system is declared out-of-control and the method will signal an alarm. We define $p_1 = \mathrm{OR}p_0/(1 + \mathrm{OR}p_0 - p_0)$, where OR is the odds ratio.

The Bernoulli CUSUM control statistic S_t is calculated as follows:

$$S_t = \max(0, S_{t-1} + (y_t - k)) \quad \text{for an increase in } p \qquad (1)$$

$$S_t = \min(0, S_{t-1} + (y_t - k)) \quad \text{for a decrease in } p \qquad (2)$$

where $k > 0$ is the reference value and $S_0 = 0$. The chart will signal if $|S_t| \geq |h|$, where h is the control limit. The performance of a CUSUM chart is evaluated by the average number of observations to signal (ANOS). When the system is in-control the ANOS (said ANOS_0) should be sufficiently large to have few false alarms. In the other hand, if the process shifts in the out-of-control state the ANOS (ANOS_1) should be as short as possible to quickly detect the shift. Formulas based on Corrected Diffusion (CD) approximation are used in order to obtain k and h, given p_1 and a desired false alarm rate expressed as ANOS_0. A simulation study (data not shown) showed that the use of the approximate formulas produces a slight overestimation of the in-control ANOS (ANOS_0) and then a reduction in the false alarm rate.

Simple Risk-Adjusted Bernoulli CUSUM

We propose a simple adjustment to the Standard Bernoulli CUSUM chart to take into account varying in-control risks $p_{0,t}$, using the following correction to (1) and (2):

$$S_t = \max(0, S_{t-1} + c_t(y_t - k_t)) \qquad (3)$$

$$S_t = \min(0, S_{t-1} + c_t(y_t - k_t)) \qquad (4)$$

where $c_t = h/h_t$ is the correction factor, k_t and h_t are respectively the specific reference value and the specific control limit for patient t. These parameters are obtained with the CD approximation formulas [3] using the specific $p_{0,t}$ and the desired OR and ANOS_0. Finally h represents the global control limit determined considering a desired overall performance level p_0. As usually, if $|S_t| \geq |h|$ the system will issue an alarm. Unlike the Steiner's method, which uses the predisposed risk distribution of an historical data set to compute the h value, the proposed procedure is not based on this predisposed risk distribution to compute k_t, h_t and h.

3 Simulation study

To illustrate the performance of the Steiner's Risk Adjusted CUSUM (RA CUSUM) and of the proposed Simple Risk Adjusted CUSUM (S-RA CUSUM) a simulation study was performed using data concerning 30-day mortality after cardiac surgery (UK, 1992-1998)[5]. The first two years data (training data set with a mortality rate equal to 0.065) were used to estimate a risk predictive logistic equation, to determine the parameters of the Steiner's RA CUSUM and to simulate three different scenarios. In the scenario A the predisposed risk distribution and the overall reference level were the same as in the training data set; in the scenario B the distribution of the predisposed risk was changed in order to obtain more patients at high-risk than at low-risk, while the overall reference level remained unchanged; finally, in the scenario C the predisposed risk distribution was not changed but the desired overall reference level was decreased to 0.02. For each scenario 25 million of Bernoulli random data were used. For predicting the risk we used the predictive logistic model proposed by Steiner et al. (2000): $\text{logit}(p_{0,t}) = -3.68 + 0.077 Parsonnet\_score_t$.

Each scenario was analyzed by the RA CUSUM designed to detect an OR = 2 in the mortality with a desired $ANOS_0$ = 9600. The used control limit h was 4.5. For each scenario we also applied the proposed S-RA CUSUM designed for the same values of OR and desired $ANOS_0$.

Results are reported in Table 1. Simulations show that in each scenario the actual $ANOS_0$ (corresponding to an OR = 1) of the RA CUSUM is very different from the desired value of 9600, while that of the S-RA CUSUM is sufficiently close to the desired one, although it results slightly more conservative. Therefore, because of the difference in $ANOS_0$ of the two methods the comparison of the out-of-control ANOSs ($ANOS_1$) between the two approaches is not warrantable.

Table 1: Comparison between the Risk-Adjusted CUSUM (RA CUSUM) and the Simple Risk-Adjusted CUSUM (S-RA CUSUM) designed to detect an OR = 2 with a desired $ANOS_0$ = 9600: actual in-control $ANOS_0$ (OR = 1) and out-of-control $ANOS_1$ (OR = 1.5, 2, 2.5).

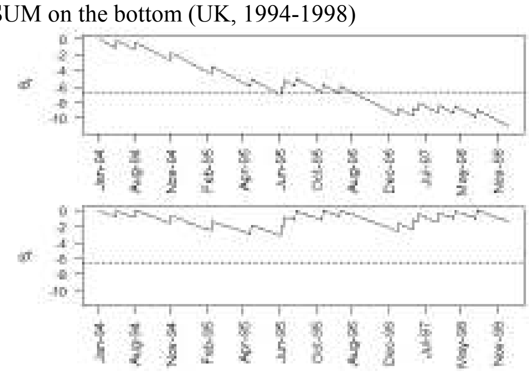
Scenario	***RA CUSUM***				***S-RA CUSUM***			
	OR				***OR***			
	1	***1.5***	***2***	***2.5***	***1***	***1.5***	***2***	***2.5***
A	7433	549.9	207.9	127.8	10879	640.8	230.7	140.1
B	2466	186.3	78.7	52.7	11465	298.5	109.5	71.4
C	20188	1560.5	580.7	350.3	10419	1256.2	506.2	308.7

4 Real data example

To better illustrate the characteristics of the proposed S-RA CUSUM, we apply the Standard Bernoulli CUSUM and the S-RA CUSUM chart to Steiner's data, considering overall performance level equal to 0.065 (global mortality rate in 1992-1993). Both the procedures are designed to detect a halving (OR = 0.5) in the odd of mortality with a desired in-control ANOS equal to 9600.

The two CUSUMs are plotted in Figure 1. The Standard CUSUM chart (on top) signals a decrease in the mortality, while the proposed simple risk-adjusted CUSUM (S-RA CUSUM) (bottom) does not signal any decrement in mortality; hence we can conclude that the surgical performance did not change during the monitored period.

Figure 1: Centre of cardiac surgery: Standard Bernoulli CUSUM on top, Simple Risk-Adjusted CUSUM on the bottom (UK, 1994-1998)



5 Discussion

Compared to the RA CUSUM the proposed Simple Risk-Adjusted CUSUM seems to be less sensitive to changes occurred in the underlying distribution of case mix and/or in the overall reference level; its performance in terms of false alarm rate is close enough to the desired value. A good performance is obtained also in case of relatively small expected number of events.

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