A decision support system for duopolies with incomplete information

Paola Vicard and Julia Mortera

Abstract We show how a symmetric game with incomplete information can be represented by an influence diagram. Object-oriented Bayesian networks allow its extension to a repeated game where the uncertainty about a further stage is modelled via a suitable Bayesian network connected to the influence diagram. Based on real cases of financial intermediation mergers a decision support system is built and its use is illustrated in different scenarios.

Key words: duopoly, incomplete information, object-oriented Bayesian networks

1 A Bayesian network representation of duopoly with incomplete information

Bayesian networks for decision support systems can incorporate both decision nodes and utility nodes [1] giving rise to an influence diagram (ID) representation. A duopoly between two firms, termed Firm1 and Firm2, can be modelled as a prisoner's dilemma (PD) which, being a symmetric game, can be represented as the ID in Figure 1. The simultaneity of the game is implemented by representing Firm1 as a random variable (oval node), and Firm2 as the decision maker (rectangular node) having two possible actions: defect (0), and cooperate (1). Firm2's utility is given in the utility node U2 (rhombus). Firm2's decision is influenced by Firm1. Firm1's associated prior probability distribution represents Firm2's subjective opinion about Firm1's behaviour.

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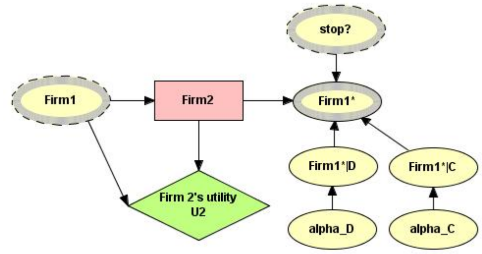

Fig. 1 Bayesian decision network for a duopoly with incomplete information.

The network in Figure 1 models one stage of a repeated PD. In what follows nodes are indicated in teletype. In each stage the game can either continue or terminate. `Firm1` and `Firm1`* have three possible states: defect (0), cooperate (1) and stop (2). Firm1* represents the behavior of Firm1 in the next stage. The uncertainty about the existence of further stages is modelled by a random node `stop?`. Node `stop?` has two states, $\{0,1\}$ according to whether the game continues or stops and has a Bernoulli distribution $Bin(1, 1-\texttt{delta})$. The parameter node `delta` is the probability that the game continues $P(\texttt{stop?} = 0)$. The parameter `delta` can either be fixed by the decision maker, or, as we will show in what follows, it can be estimated from a statistical model.

Tit for tat (TFT) is a simple common strategy in which Firm1 begins by cooperating and cooperates as long as Firm2 cooperates, and defects otherwise. However, usually there is uncertainty about the type of rival that a firm is going to face and we need to include a set of potential strategies for Firm1. The network in Figure 1 models the general class of strategies under *incomplete information*, where nodes `Firm1`*`|D` and `Firm1`*`|C`, have Bernoulli distributions with parameter nodes `alpha_D` and `alpha_C`. `Firm1`* depends on `Firm2` and if `Firm2` defects `Firm1`* is `Firm1`*`|D`, else `Firm1`*`|C`. In this way we represent Firm2's subjective opinions about Firm1's behaviour in each stage of a repeated game. The conditional probability distribution of Firm1* reflects Firm2's uncertainty about its opponent. If Firm2 believes Firm1 to be "altruistic" it can expect Firm1 to cooperate (with a probability $\alpha_D > 0$) even if it defected in the previous stage. On the other hand, if Firm2 believes Firm1 to be "egoistic", then it expects Firm1 to cooperate with probability $\alpha_C < 1$ even if it cooperated in the previous stage.

2 Application to mergers of financial intermediation firms

Here we examine *financial intermediation* economic sector and model the merging decision process of two firms under the Antitrust Authority's (AA) control activity. The AA's decision to intervene, in order to prevent anticompetitive behaviour, corresponds to the node `Stop?` in Figure 1. The decision problem is modelled by means of object-oriented Bayesian networks (OOBN) [2] as shown in Figure 2. The networks **Duopoly_1**, **Duopoly_2** and **AA_1**, **AA_2** are instances of the networks

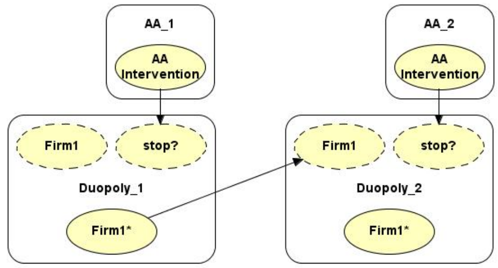

Fig. 2 Object-oriented Bayesian network for a decision problem with uncertainty about the number of stages.

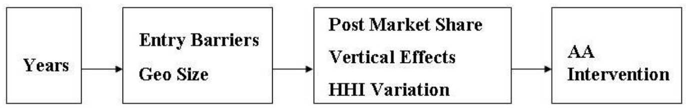

Fig. 3 Logical constraints for learning the AA intervention process. If there is a relation between two variables in different boxes, it must have the direction specified in this chain.

in Figure 1 and 4, respectively. In each game stage the node `AA Intervention` (modelling the AAs decision to intervene) is directly linked to the node `Stop?` by an identity relation carrying the information from the market of interest (modelled in AA networks) into the merger decision problem (modelled in the Duopoly networks). The AA network is estimated from the data collected by the Italian Antitrust Authority concerning all cases examined from 1991 to 2003 [3] using the *Necessary Path Condition* (NPC) algorithm [4]. We thus took into account the logical constraints (such as assignment/ban of a specific edge direction between nodes) shown in Figure 3. The estimated network in Figure 4 represents the association structure and the marginal probability distributions for each variable (node). The AAs decision to intervene is directly influenced by the reference period (`Years`), the presence/absence of entry barriers (`Entry Barriers`), the post merger market share (`Post Market Share`) and the variation in the Herfindahl-Hirshman concentration Index (`HHI Variation`).

Once the model for AA has been estimated (Figure 4) and the global decision problem has been modelled as a repeated game with uncertainty about the number of stages (Figure 2), various scenarios can be simulated in a mouse-click time[1]. We analyse how cooperative behaviour varies with Firm2's opinion about its opponent (Firm1) and with factors influencing the AA's decision. As an example, two scenarios regarding the relevant market are considered: no evidence and evidence E={`Post Market Share`$\geq$ 40% and `Vertical Effects`= Yes}. These two scenarios are analysed for three possible cases concerning Firm2's opinion about its rival Firm1. Table 1 gives the expected utilities for cooperating and defecting for each scenario/case. When no information on the reference market is available, Firm2's optimal decision is to cooperate under the TFT strategy (when $\alpha_C = 1$ and $\alpha_D = 0$), and, when Firm2 believes that Firm1 cooperates with proba-

[1] We use the software HUGIN to build and estimate the networks and to implement our examples.

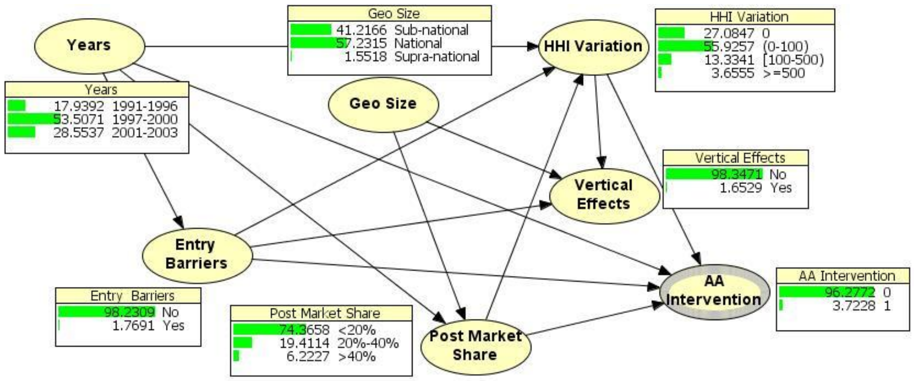

Fig. 4 Bayesian network to model AA intervention.

bility $\alpha_C = 0.8$ ($\alpha_D = 0.2$) if Firm2 cooperates (defects). This is due to the fact that the probability of AA intervention is very small (0.0372). Only when $\alpha_C = 0.6$ and $\alpha_D = 0.4$, Firm2's expected utility is maximised by defecting. Instead, the probability AA intervenes is large (0.8376) under evidence E, and consequently Firm2's optimal decision is to defect in all cases considered here.

Table 1 Firm2's expected utility for different values of α_C and α_D and for the TFT strategy, without evidence and with evidence E.

		without evidence		with evidence E	
α_C	α_D	$\mathbb{E}[u(\text{Defect})]$	$\mathbb{E}[u(\text{Cooperate})]$	$\mathbb{E}[u(\text{Defect})\|E]$	$\mathbb{E}[u(\text{Cooperate})\|E]$
0.8	0.2	104	161	80	64
0.6	0.4	133	132	85	60
TFT		150	244	150	124

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