Bayesian Unit Root Tests: a Monte Carlo Study

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Abstract In this paper we want to shed some more light on an old debate about classical and Bayesian unit root tests. Within this discussion, Koop (1992) proposed a set of Bayesian unit root tests that aims at overcoming many of the traditional unit root tests drawbacks, but do not depend on the researcher's prior opinion.
Our aim is to further investigate, via simulations, Koop's Bayesian unit root tests after a few years of silence, also in comparative terms with regards to classical test.

1 Introduction

In the past few decades, the econometrics literature presented tests to identify unit roots. The greatest advances have been made by the frequentist tradition, but in the late 1980's Bayesians started also the investigation on this topic and presented interesting alternatives.

The first proposal of Bayesian unit root test, based on a flat prior, is in Sims (1988). Interestingly, following Sims (1988), DeJong and Whiteman (1991) obtained results contrary to the unit roots findings obtained by Nelson and Plosser (1982), based on the ADF test.

Classical econometricians, e.g. (Stock, 1991), argue that disagreement over priors leads to different posterior and different results. Moreover, flat-prior analysis of unit root can produce rather biased estimators and interval estimates with incorrect asymptotic confidence levels. The frequentist critics received an answer in Phillips (1991) who derived the Jeffreys prior as an alternative to the flat-prior and this was the starting point of a heat debate (Bauwens *et al*., 1999).

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Within this discussion, Koop (1992) proposes a set of Bayesian unit root tests that while not depending on the researcher's prior opinion, aims at overcoming many of the traditional unit root tests drawbacks (in particular low power). Recently, Diniz *et al* (2011) propose a Full Bayesian Significance Test to test sharp hypotheses, in particular the unit root sharp hypothesis, based on a sophisticated algorithm.

In this paper we intend to shed some more light on Koop's Bayesian test for unit root via a Monte Carlo experiment that compares its performance with the ADF and Phillips-Perron (PP) classical tests for a number of specifications.

2 Koop's objective test for unit root

The logic of Bayesian tests for unit root is, in general, based on the posterior odds ratio for comparing hypothesis:

$$K_{12} = \frac{P(H_1)P(Data \mid H_1)}{P(H_2)P(Data \mid H_2)}$$

The idea is that the probability that the data would have occurred if H_1 were true is compared with the probability that the data would have occurred if H_2 were true.

Instead of using a noninformative prior that, as Phillips (1991) showed, is likely to give unreliable results since sharp null are strongly favoured, Koop (1992) considers the use of reference priors. These appear to be very useful since they allow automatically eliciting priors when the researcher has little prior information or does not intend to impose information at all. In particular, Koop choices two reference priors: the ZS-prior (Zellner and Siow, 1980) and the g-prior (Zellner, 1986).

The ZS-prior has been proposed for testing an exact linear restriction. The posterior odds ratio comparing the nested to the larger model is:

$$K_{12} = \frac{b(v_1/2)^{r/2}}{[1 + rF/v_1]^{(v_1-1)/2}}$$

where $b=\pi^{1/2}/\Gamma[(r+1)/2]$, F is the test statistic for restrictions, r is the number of tested restrictions, N is the series length, k the number of regressors in the larger model, $v_1=N-k$. The ZS-prior has the form of a Cauchy distribution, centered over the restriction being tested without subjective input from the researcher.

The g-prior works for more general hypotheses, H1: $y=X_1\beta_1+\varepsilon_1$, H2: $y=X_2\beta_2+\varepsilon_2$ and in this case the posterior odds ratio comparing model 1 to 2 is

$$K_{12} = \left(\frac{g}{1+g}\right)^{(k/2)} \left[\frac{1+\left(\frac{g}{1+g}\right)\frac{k}{v}F_2}{1+\frac{k}{v}F_1}\right]$$

where F_1 and F_2 are test statistics for restrictions, k the number of regressors in the larger model, $v=N-k$.

The g-prior requires the specification of prior location and parameter g. As for the prior location, it is specified by Koop (1992) as a random walk. The g parameter ($0<g<1$) is directly connected to the degree of informativeness of the prior, as the prior precision for β_i is $gX_i'X_i$. Note that as g gets bigger, the prior becomes more informative.

Conditionally to σ^2, the posterior covariance of β_i is $\frac{(X_i X_i')\sigma^2}{1+g}$. If g=0 the prior is noninformative, whereas if g=1 the prior is equally informative to the data.

3 Monte Carlo study

A Monte Carlo experiment[1] has been conducted by Koop (1992) to assess the finite sample performance of the Bayesian tests with ZS-prior and g-prior. The experiment is done within the framework of the distinction between trend stationary (TS) and difference stationary (DS) model.

As for the interpretation, the size is one minus the proportion of times the probability of DS hypothesis is greater than the probability of the TS hypothesis in repeated samples drawn from DS models. Power is the proportion of times the probability of the TS hypothesis exceeds that of the DS hypothesis in repeated samples drawn from a specific TS model.

Koop's simulation design has been realized with series length equal to 100. We believe that the results might be affected by the series length. So in this work we firstly update Koop's results, by considering the same experiment schemes with increasing series length (N=100, 200, 300, 400, 500). Moreover, we compare the finite sample results of Koop's objective tests (g-prior and ZS) with the classical ADF and PP tests.

Along the lines of Koop (1992), with increasing order of complexity, we carried out 3 experiments, where the coefficients have been attributed from the OLS estimates of real GNP data, H1 is the DS hypothesis and H2 is the TS hypothesis.

1) H1: $\Delta y_t = 0.02985 + u_t$

H2: $y_t = 0.60673 + 0.87225\, y_{t-1} + 0.00435\, t + u_t$

2) H1: $\Delta y_t = 0.01947 + 0.34118\, \Delta y_{t-1} + u_t$

H2: $y_t = 0.81906 + 1.24355\, y_{t-1} - 0.41889\, y_{t-2} + 0.00565\, t + u_t$

3) H1: $\Delta y_t = 0.02119 + 0.36925\, \Delta y_{t-1} - 0.082556\, \Delta y_{t-2} + u_t$

H2: $y_t = 0.884 + 1.218\, y_{t-1} - 0.354\, y_{t-2} - 0.052\, y_{t-3} + 0.006\, t + u_t$

Moreover, for what concerns the study of the power and focussing in this few words of description only on AR(1) alternatives of the first experiment, we conducted the experiment for increasing value of the autoregressive parameter[2] ρ=0.75, 0.80, 0.85, 0.90, 0.95, 0.99 and various initial displacements $x^* = (y_0 - \mu)/\sigma^2$, in particular x^*=0, 4, 8, as in Koop (1992). Another very relevant issue is the choice of g that has been done by Koop on the basis of a sensitivity analysis. According to Koop's results (confirmed also by our simulations, conducted with increasing series length and available upon request) a good choice, compatible with classical tests results is g=0.002. Finally, to compute the posterior odds ratio, linear constraints need to be specified and tested[3].

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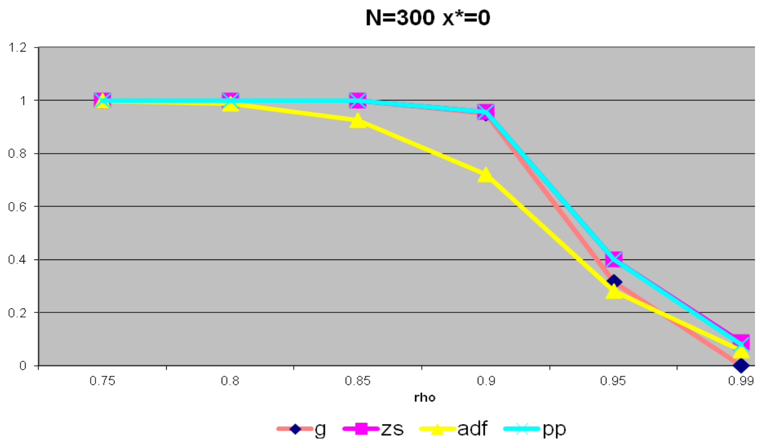
[1] Ahking (2009) also carried out a Monte Carlo study on this topic but he considered the Zellner-Siow prior only. Moreover he focuses only on a limited number of data generating processes.

[2] For experiment n.2 and n.3 this is naturally extended.

[3] For space reasons, this is not the text, but it available upon request by authors.

In Figure 1 some selected size results (relative to *N*=300, experiment 1) are reported. As we can notice. Both Koop's tests have good power performance, in line with PP test, definitely better than ADF test. This is confirmed also for the other power results. Size results, that for space reasons cannot be shown, are also satisfactory: Koop's test manage to be in line with ADF and PP. Finally, as *N* increases (results not reported for space reasons) both Koop's test improves. In particular, there is an improvement in the power of the g-prior and the size of the ZS prior test.

Figure 1: *Power results – Experiment 1*



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