

# Contributions from income components to Zenga's point and synthetic inequality measures: an application to EU countries.

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**Sommario** In this work we analyze contributions from income components to Zenga's point and synthetic inequality measures in the distribution of household income in EU countries. The contributions are computed according to a decomposition rule recently proposed in [3]. The empirical results obtained in this work confirm the usefulness of this approach in understanding the sources of inequality in the distribution of household income.

**Key words:** Zenga synthetic inequality index, upper and lower means, point inequality measures, decomposition by income components

## 1 Introduction

Recently Zenga [2] proposed a new inequality index based on upper and lower group means. Given  $n$  households whose incomes are given by

$$0 \leq y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(n)}, \quad y_{(n)} > 0, \quad (1)$$

Zenga's inequality measure is defined as

$$I = \frac{1}{n} \sum_{i=1}^n I_i, \quad \text{where} \quad I_i = \frac{M_i^+ - \bar{M}_i}{M_i^+}, \quad i = 1, 2, \dots, n, \quad (2)$$

and

$$\bar{M}_i = \frac{1}{i} \sum_{t=1}^i y_{(t)}, \quad i = 1, 2, \dots, n, \quad M_i^+ = \begin{cases} \frac{1}{n-i} \sum_{t=i+1}^n y_{(t)}, & \text{if } i = 1, 2, \dots, n-1, \\ y_{(n)}, & \text{if } i = n \end{cases}$$

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are the so called lower and upper means, respectively. Notice that each  $I_i$  is a point inequality measure, which compares the mean income of the  $i$  poorest households (the  $i$ -th lower group) with that of the remaining household population (the  $i$ -th upper group).

Assume now that each household income in (1) is the sum of  $c$  income components and let  ${}_j\bar{M}'_i$  and  ${}_j\bar{M}'_i$  be the means of income component  $j$  in the  $i$ -th lower and upper group, respectively. The point inequality measures may then be written as

$$I_i = \frac{{}_i\bar{M}_i - {}_i\bar{M}_i}{{}_i\bar{M}_i} = \sum_{j=1}^c \frac{{}_j\bar{M}'_i - {}_j\bar{M}'_i}{{}_i\bar{M}_i}, \quad i = 1, 2, \dots, n, \quad (3)$$

and

$${}_jB_i = \frac{{}_j\bar{M}'_i - {}_j\bar{M}'_i}{{}_i\bar{M}_i}$$

may be viewed as the contribution from income component  $j$  to inequality between the  $i$ -th lower and upper groups. Dividing that contribution by  $I_i$  yields

$${}_j\beta_i = \frac{{}_jB_i}{I_i} = \frac{{}_j\bar{M}'_i - {}_j\bar{M}'_i}{{}_i\bar{M}_i - {}_i\bar{M}_i},$$

which is the relative contribution from income component  $j$  to the difference between the  $i$ -th lower and upper mean of overall household income.

Given the decomposition rule for the point inequality measures  $I_i$ , the contribution from income component  $j$  to the synthetic  $I$  index in (2) may be assessed through the average

$${}_jB = \frac{1}{n} \sum_{i=1}^n {}_jB_i,$$

since, as may be easily verified,  $I = \sum_{j=1}^c {}_jB$ . Dividing  ${}_jB$  by  $I$  yields the relative contribution from income component  $j$  to the synthetic  $I$  index, which will be denoted by  ${}_j\beta$ .

From the definitions of the contributions, one should expect that components whose share  ${}_j\gamma$  on total population income is large, should have large contributions to inequality as well. It seems therefore useful to compare the contributions with those under the hypothesis that in each single household the share of income from each component equals the share of that component on total population income. In the following we shall refer to this hypothesis as the *scale transformation hypothesis*. It is not hard to show that  ${}_jB_i = {}_j\gamma I_i$ ,  ${}_jB = {}_j\gamma I$  and thus  ${}_j\beta_i = {}_j\beta = {}_j\gamma$  for  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, c$  under the scale transformation hypothesis.

## 2 Empirical analysis of ECHP survey data

In this work we analyze household income from the 2001 wave of the ECHP survey [1] and compute contributions from income components to inequality as measured by Zenga's synthetic  $I$  index. The income components we consider are *net of taxes*. They are wage and salary income ( $j = 1$ ), self-employment income ( $j = 2$ ), social transfers ( $j = 3$ ) and other income components ( $j = 4$ ) that include capital income, income from property and rents and private transfers. For the definitions of these components we refer to the ECHP documentation on EuroStat's web-page.

Table 1 reports some summary information from each country. The countries are ordered according to the value of Zenga's synthetic  $I$  index from Luxembourg, the country where inequality is lowest ( $I = 0.626$ ), to Portugal, the country with highest inequality ( $I = 0.753$ ).

Analyzing the shares of the income components on total population income, we immediately notice some common features among the 15 countries: wages and salary account for most of population income in all countries, with shares ranging from 46.1% in Greece to 69.2% in Denmark. It follows social transfers, with shares ranging from 20.2% in Ireland to 32.3% in Austria. The share of self-employment income may vary considerably from country to country and it usually exceeds the aggregate share of the other income components. In fact, self-employment income accounts for barely 1.9% of population income in Sweden and its share increases to 23% in Greece, while the aggregate share of the other income sources exceeds 5% only in UK, Greece and Belgium and reaches its maximum value of 7.9% in Belgium.

As for the relative contributions  ${}_j\beta$  (table 2), we note that wages and salary account, as expected, for most of inequality in all the 15 countries. Their share on  $I$  exceeds the share on population income by values ranging from 7.6% in Finland up to 23.3% in Sweden. Self-employment income also exhibits larger contributions to inequality than under the scale transformation hypothesis (except for Sweden), although the differences are smaller than for wage and salary income. They range from  $-0.8\%$  in Sweden to about 10% in Ireland. Social transfers, on the other hand, contribute less to inequality than under the scale transformation hypothesis, with negative differences ranging from 15.6% in Portugal to 25.8% in Austria. Finally, we note that the aggregate contribution from the other income sources is usually quite similar to their share on total population income, except for Finland, where it is almost 6% higher.

## Riferimenti bibliografici

1. EuroStat: European Community Household Panel (ECHP UDB - version of December 2003)
2. Zenga, M.M.: Inequality curve and inequality index based on the ratios between lower and upper arithmetic means. *Statistica & applicazioni* **5**(1), 3–27 (2007)
3. Zenga, M., Radaelli, P. and Zenga, M.: Decomposition of Zenga's inequality index by sources. Working paper n. 216, DiMeQUANT, Università degli Studi di Milano-Bicocca (2011)

**Tabella 1** General information about EU-countries as from the ECHP Survey

Country	Pop. (mln)	hh.size	$n$	Median <sup>b</sup> $y_{(i)}$	Mean <sup>b</sup> $y_{(i)}$	$I$	$1\gamma$	$2\gamma$	$3\gamma$	$4\gamma$
LUX	0.433	2.52	2428	39435	45736	0.626	0.641	0.041	0.272	0.046
SWE	8.663	1.89	5085	24711	26293	0.642	0.644	0.019	0.311	0.026
GER	81.569	2.16	5559	25742	28060	0.646	0.609	0.078	0.271	0.043
NET	15.773	2.29	4824	22842	25046	0.657	0.669	0.040	0.266	0.025
AUS	7.986	2.42	2535	25722	28823	0.664	0.584	0.066	0.323	0.028
DEN	5.368	2.19	2279	28511	32502	0.665	0.692	0.051	0.231	0.026
ITA	57.388	2.61	5525	17676	20634	0.686	0.527	0.169	0.270	0.035
FRA	57.949	2.36	5247	23908	28292	0.692	0.603	0.068	0.288	0.042
BEL	10.263	2.40	2342	23798	29361	0.710	0.587	0.075	0.260	0.079
SPA	39.137	2.95	4950	15159	18431	0.719	0.588	0.123	0.258	0.032
UK	59.063	2.31	4779	26134	30973	0.721	0.651	0.080	0.218	0.052
GRE	10.354	2.59	3895	11539	14073	0.733	0.461	0.230	0.255	0.054
IRE	3.839	2.97	1757	25400	29919	0.738	0.601	0.172	0.202	0.026
FIN	5.120	2.15	3106	30985	37628	0.747	0.625	0.101	0.211	0.063
POR	10.024	2.96	4588	10232	12705	0.753	0.601	0.110	0.266	0.023

<sup>a</sup> The average household size is obtained dividing the population by the number of households. The ECHP provides both figures for each country.

<sup>b</sup> The medians and the means of total household income are expressed in Euro. They have been obtained using the fixed conversion rates for Germany, Denmark, Netherlands, Luxembourg, France, UK, Ireland, Italy, Greece, Spain, Portugal and Austria and using the conversion rate for the year 2001 as given by the ECHP for Belgium, Finland and Sweden.

**Tabella 2** Relative contributions to Zenga's synthetic  $I$  index from income components

Country	$I$	$1\beta$	$2\beta$	$3\beta$	$4\beta$	$1\beta - 1\gamma$	$2\beta - 2\gamma$	$3\beta - 3\gamma$	$4\beta - 4\gamma$
LUX	0.626	0.798	0.090	0.035	0.077	0.157	0.049	-0.237	0.031
SWE	0.642	0.877	0.011	0.059	0.053	0.233	-0.008	-0.252	0.026
GER	0.646	0.729	0.170	0.041	0.060	0.120	0.093	-0.230	0.017
NET	0.657	0.841	0.079	0.051	0.029	0.172	0.039	-0.215	0.004
AUS	0.664	0.805	0.103	0.065	0.027	0.222	0.037	-0.258	-0.001
DEN	0.665	0.875	0.111	-0.005	0.019	0.183	0.060	-0.236	-0.007
ITA	0.686	0.625	0.232	0.097	0.046	0.099	0.063	-0.173	0.011
FRA	0.692	0.754	0.118	0.092	0.036	0.151	0.050	-0.196	-0.005
BEL	0.710	0.710	0.155	0.002	0.133	0.123	0.080	-0.258	0.055
SPA	0.719	0.726	0.177	0.059	0.038	0.138	0.054	-0.199	0.007
UK	0.721	0.811	0.128	0.000	0.061	0.160	0.048	-0.217	0.009
GRE	0.733	0.604	0.258	0.072	0.067	0.143	0.028	-0.184	0.013
IRE	0.738	0.714	0.269	-0.013	0.030	0.114	0.097	-0.215	0.004
FIN	0.747	0.701	0.136	0.043	0.120	0.076	0.035	-0.168	0.057
POR	0.753	0.724	0.128	0.110	0.038	0.123	0.018	-0.156	0.015