Equivalence scales, inflation, and PPP: a unique (and simple) approach to estimation

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Abstract We assume that the standard of living of a household is a latent variable, the proper manifest variables of which must satisfy an explicit condition of consistency: for every given household dimension, they must vary monotonically with available resources (income, or total expenses). On the basis of several, properly selected, manifest variables, we form clusters of households. Within clusters, where, by construction, the standard of living is similar, we find households (a) of different dimension, (b) from different regions, or (c) observed in different years. Within each cluster, we can therefore calculate several index numbers: (a) ES, or Equivalence Scales, (b) PPP, or Purchasing Power Parities, and (c) measures of inflation. Averaging over the various cluster-specific index numbers we obtain the synthetic measures we are looking for (ES, PPP, inflation). We show that this simple, non-parametric approach is robust and flexible, can be applied to a variety of datasets (here: Bank of Italy’s SHIW, 2004-08), and solves a number of theoretical and practical estimation problems.

1 From Utility to Equivalence Scales (Inflation, and PPP)

Economists define “utility” $u$ (or standard of living) as a monotonically increasing function of the goods and services that a consumer can buy, so that $u=u(q)$, with $u'>0$, where $q$ is a vector of quantities. Each consumer will buy as much as possible, subject to a budget constraint, given by disposable income $Y$ (or, sometimes, total outlays $X$), and prices ($p$, once again, a vector).

When the question is about households’ (not individuals’) utility, the notation becomes $h_u=q(h_d)=u=Y, p, h_d)$, where the vector $h_d$ represents households (demographic) characteristics, frequently just the number of its members. Conceptual difficulties increase in this case, for at least two reasons: the idea of a common utility for
all household members may be questioned, and family characteristics may impact one’s utility directly. Ignoring both objections, as we will do here, one can force utility to be the same in two demographically different cases (\(u=r\); \(r=reference\)), and thus find the level of income \(Y\) that makes household \(h\) as well off as household \(r\), with income \(Y\). The ratio \(Y/Y\) is the equivalence scale, that is the relative income that "compensates" a household of type \(r\) for a change in its demographic characteristics, from \(d\) to \(d\). (Prices \(p\) need not be known if they can be assumed to be the same for both households.)

The first original contribution of this paper (but see also De Santis and Maltagliati, 2010) is that we let two more variables appear as “household characteristics”: year of observation \(t\) and area of residence \(a\) (region, country, etc.). Therefore, the ratio \(Y/Y\) that we will later compute on sample data is an estimate of the price increase between \(t\) and \(t\). Similarly, the ratio \(Y/Y\) is an estimate of the average price ratio in the two areas considered, \(a\) and \(a\), or \(PPP\) (Purchasing Power Parity). In both cases, as with the estimation of \(ES\), the crucial assumption is that households belonging to the same cluster enjoy the same standard of living. The remarkable feature, here, is that we can estimate inflation and \(PPP\) without knowing prices.

2 The Elusive Notion of Utility: Economics vs. Statistics

The main difficulty with the economic approach to the estimation of equivalence scales is that, even with highly simplifying assumptions, deriving utility from general theoretical assertions and comparing it across households is a daunting task. Empirical results frequently prove senseless, or unstable, or both.

The second contribution of this paper is a suggestion as to how to overcome these difficulties. Let us assume that the standard of living is a latent variable that is correlated with “consistent” manifest variables, where consistent means that, for any given household composition (=number of members), these variables must vary monotonically with income.

If our indicators (=manifest variables) were related only to the standard of living (latent variable), and to nothing else, each of it would suffice - and, incidentally, Engel’s method falls squarely into this type of approach, because the food share is frequently (but erroneously) believed to depend only on how well off a household is. But we improve over Engel and his epigones in two respects: a) we use several manifest variables to signal utility, not just one (food share); and b) we do not try to measure utility (cardinal approach), merely to identify groups (clusters) of households where

1 See e.g. Browning et al. (1994). We will instead assume that the standard of living is (basically) the same within families. In this paper, we will use the terms “family” and “household” interchangeably.

2 For instance, those who get married or have children may need to spend less on social life precisely because they have a family, who satisfies their need for company, at least in part.

3 Economists who want to measure economies of household scale face unpalatable choices. They can use the atheoretical Engel method, which works but makes no sense ... or they can try a method ... that makes sense but does not work” (Gibson 2002: 357). And: "In general, there is no accepted method for determining equivalence scales, and no equivalence scale is recommended by the OECD for general use" OECD (2008).

4 Or total outlays, depending on which is available. The derivative can be either negative (if the good is a necessity, like food share: it decreases as income increases) or positive (if the good is a luxury: it increases with income), over the entire range of observed income, separately for each household composition.
utility is (or can be assumed to be) more or less the same, despite possible variations in households’ dimension, resources, region and year of observation. This is consistent with an ordinal approach to the study of utility, normally accepted in literature.

Resources (income or total expense) are not among the manifest variables that we consider for clustering. But once we have our clusters, within each of it we can calculate the average resources of the reference household ($Y_r$) and of any other household typology ($Y_h$). The (assumed) equality of utility within clusters is precisely what permits us to claim that the ratio $Y_r/Y_h$ is an estimate of cluster-specific ES, PPP, or inflation, depending on what differentiates the compared households. Averaging over all clusters, we arrive at an estimate of the more general measures we are looking for.

### 3 A few empirical results

After having applied this approach to Istat (micro) consumption data of 2003 to 2008 (De Santis and Maltagliati 2010), in this paper we apply it to SHIW data (Survey of Household Income and Wealth, by Bankitalia), more or less on the same period (2004, 2006, and 2008), but on a smaller sample (about 23.4 thousand households, of 1 to 5 members, as against 128 thousand, with Istat data), and on a totally different set of manifest variables - which, in a few cases, include subjective ones

| Table 1: Equivalence scales, inflation and PPP estimated on Istat and SHIW data. Italy. |
|---|---|---|---|---|---|---|
| Hhld Dimen. | Equivalence scale | Inflation (Base 2004=1) | PPP in Italy |
| | Istat | SHIW | Square root | | Istat | SHIW | Istat | SHIW |
| 1 | 1 | 1 | 1 | 2003 | 0.959 | 0.978 | NW | 1 | 1 |
| 2 | 1.263 | 1.475 | 1.414 | 2004 | 1 | 1 | 1 | NE | 1.001 | 0.968 |
| 3 | 1.526 | 1.826 | 1.732 | 2005 | 1.018 | (1.04) | 1.019 | Center | 0.989 | 1.032 |
| 4 | 1.672 | 2.049 | 2.000 | 2006 | 1.040 | 1.085 | 1.041 | South | 0.829 | 0.759 |
| 5 | 1.786 | 2.160 | 2.236 | 2007 | 1.059 | (1.10) | 1.060 | Islands | 0.779 | 0.745 |
| | | | | 2008 | 1.077 | 1.110 | 1.095 |


Table 1 shows that, despite the ample differences in the data sets and in the manifest variables, the method yields comparable results on both Istat and SHIW data. The resulting equivalence scale is not far from the OECD (2008) square root one - which is now “fashionable”, but for which, to the best of our knowledge, nobody ever produced supporting empirical or theoretical evidence. PPP are reasonable, and comparable to those proposed by Cannari and Iuzzolino (2009). Our estimates of inflation are relatively poor in the first period (2004-2006) but good later on (up to 2010 - preliminary and not shown here): the apparent anomaly of 2006 data needs further investigation. Note, however, that both PPP and inflation are estimated without knowing prices.

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1 Manifest variables for SHIW are: food share; subjectively declared economic conditions; some holidays in year (dichotomous); holidays where? (hotel, house, tent, ...); subjectively perceived poverty line; share of income saved. Manifest variables for Istat are listed in De Santis and Maltagliati (2010).
4 Discussion

In both the applications that we tried (on Istat and SHIW data) we had to face the non-negligible problem of what is the “best” way to proceed: what clustering criterion to choose, how many clusters to form, whether to stratify by geographical region within clusters, whether to assume a given inflation (and inflate monetary values accordingly) or to try to estimate it (and keep monetary values at their original level), etc. Unfortunately, results a sensitive to each of these decisions (not shown here). Our choices (Ward method, 5 regions, 20 to 100 clusters; with and without re-evaluation of monetary values) may be questioned and we cannot offer any solid theoretical justification for them: merely empirically reasonable results, that seem to strike a satisfactory balance between number of observations, within-cluster homogeneity, and consistency (as the number of clusters increases).

But the method has some strengths: theoretically, it simply requires that the research identifies a set of proper manifest variables for the abstract notion of “utility”. Statistically, it is a non-parametric approach that unifies the estimation of equivalence scales, inflation and PPP, proves empirically easy to implement on a variety of sources, and, overall, gives consistent, robust and reasonable results, both on income data (not shown here) and on consumption data (De Santis and Maltagliati, 2010).

References