NON-AGGREGATIVE ASSESSMENT OF SUBJECTIVE WELL-BEING

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ABSTRACT. The assessment of well-being in a multidimensional framework is usually based on the construction of composite indicators. This classical approach has many drawbacks, due to its aggregative/compensative nature and to the inconsistencies arising when ordinal variables are dealt with. In this paper, we show how it is possible to derive a synthetic assessment of well-being out of many ordinal variables, involving no aggregation procedures. This task is accomplished using partial order, which provides all the basic tools for tackling multidimensional ordinal datasets in a consistent way. This new methodology is applied to data pertaining to subjective well-being in Italy, for year 2010. A comparison of well-being levels across regions and for different subpopulations is presented, getting interesting insights into the perceived quality-of-life in Italian society.

Keywords: Well-being, Partial order, Evaluation.

1. INTRODUCTION

This paper discusses a new statistical methodology for the construction of social indicators, based on ordinal data, providing also an application to subjective well-being data, pertaining to Italian regions. The most remarkable and distinctive feature of the proposed methodology is that only the ordinal properties of the data are exploited in indicators construction, and no variable aggregation is performed. This neatly differentiates the methodology from mainstream approaches, which are aggregative and compensative in nature and address ordinal variables using tools designed for quantitative data. In the latter case, the construction of synthetic indicators is mainly pursued through dimensionality-reduction tools which exploit correlations among variables to reduce data complexity. In practice, this often leads to building linear combinations of variables, either according to some optimality criterion, as in principal component analysis and structural equation models, or more heuristically, as in the widespread composite indicators approach. Drawing upon correlations and aggregation prevents these approaches to be directly applied to ordinal data, which in fact are turned into numerical figures prior to the statistical analysis. In many cases, ordinal scores are even interpreted in cardinal terms and directly treated as numbers, as often in social surveys based on Likert scales. Results obtained that way are arguable and difficult to interpret. Realizing these issues, the methodology discussed in this paper proposes a way to build synthetic indicators out of ordinal data without variable aggregation. This result is achieved using concepts and tools from Partially Ordered Set theory (poset theory, for short), instead of sticking to classical data analysis, based on linear algebra. In the poset setting, evaluation is addressed as a problem of comparison between multidimensional socio-economic profiles, rather than as a problem of turning multidimensional profiles directly into numerical scores, to make them comparable. This change of perspective leads to a formal framework that proves more consistent than those based on aggregative approaches.

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2. Partially ordered sets: basic definitions

A partially ordered set (or a *poset*) $P = (X, \leq)$ is a set X equipped with a partial order relation $\leq$, that is a binary relation satisfying the properties of *reflexivity*, *antisymmetry* and *transitivity* (Davey and Priestley, 2002; Neggers and Kim, 1988):

(1) $x \leq x$ for all $x \in X$ (reflexivity);
(2) if $x \leq y$ and $y \leq x$ then $x = y$, $x, y \in X$ (antisymmetry);
(3) if $x \leq y$ and $y \leq z$, then $x \leq z$, $x, y, z \in X$ (transitivity).

If $x \leq y$ or $y \leq x$, then x and y are called *comparable*, otherwise they are said to be *incomparable* (written $x \,||\, y$). A partial order P where any two elements are comparable is called a *chain* or a *linear order*. On the contrary, if any two elements of P are incomparable, then P is called an *antichain*. A finite poset P (i.e. a poset over a finite set) can be easily depicted by means of a *Hasse diagram* (Davey and Priestley, 2002; Patil and Taillie, 2004), which is a particular kind of directed graph, drawn according to the following two rules: (i) if $s \leq t$, then node t is placed above node s; (ii) if $s \leq t$ and there is no other element w such that $s \leq w \leq t$ (i.e. if t *covers* s), then an edge is inserted linking node s to node t. By transitivity, $s \leq t$ (or $t \leq s$) in P, if and only if there is a path in the Hasse diagram linking the corresponding nodes; otherwise, s and t are incomparable. Examples of Hasse diagrams are reported in Figure 1. An *upset* U of a poset P is a subset of P such that if $x \in U$ and $x \leq z$, then $z \in U$. In a finite poset P, it can be shown that given an upset U there is always a finite antichain $\underline{u} \subseteq P$ such that $z \in U$ if and only if $u \leq z$ for at least one element $u \in \underline{u}$. The upset is said to be *generated* by $\underline{u}$, written $U = \underline{u}\uparrow$. The subset $\{x, t, u, v\}$ of poset (1) in Figure 1 is an upset, generated by the antichain $\{u, v\}$. Similarly, a *downset* of P is a subset I such that if $x \in I$ and $y \leq x$, then $y \in I$. An *extension* of a poset P is a partial order defined on the same set X as P, whose set of comparabilities comprises that of P. A *linear extension* of a poset P is an extension of P that is also a linear order. Poset (2) of Figure 1 is a linear extension of poset (1) and, trivially, of the antichain (3). A fundamental theorem of partial order theory states that the set of linear extensions of a finite poset P uniquely identifies P (Neggers and Kim, 1988).

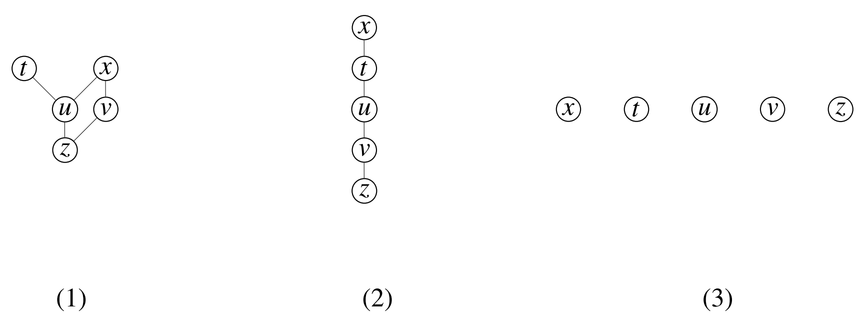

FIGURE 1. Hasse diagrams of a poset (1), a chain (2) and an antichain (3).

3. Subjective well-being data

The dataset we refer to in this paper is extracted from the Istat survey entitled "Indagine Multiscopo", pertaining to year 2010. Here we focus on a set of three variables pertaining to subjective well-being, namely: satisfaction of personal economic situation (v_1), satisfaction of personal health status (v_2) and satisfaction of personal labour situation (v_3)[2]. All of the variables are recorded on a 4-degree Likert scale, ranging from 1 (best situation) to 4 (worse situation). Data refers to Italian regions and to males and females separately. Data have been made available within a a cooperation between University of Florence and Istat[3], entitled "Quality of life in Italy: an assessment through data from Multi-purpose Survey about families. Aspects of daily life" .

4. Poset description of subjective well-being data

Partial order theory allows for a very natural and effective way of representing multidimensional ordinal datasets. Let us consider the group of subjective well-being dimensions, introduced in the previous Section. Each individual in the population is assigned a sequence $\boldsymbol{p} = p_1p_2p_3$ ($p_i = 1,\ldots,4,\ i = 1,2,3$) of scores on v_1, v_2 and v_3, which in the sequel will be called a (subjective well-being) *profile*. The number of possible profiles is $4^3 = 64$ and the set of all of them will be denoted P. Well-being profiles can be (partially) ordered according to the following definition

Definition 4.1. *Profile $\boldsymbol{p}_a$ is more or equally unsatisfied than profile $\boldsymbol{p}_b$ (written $\boldsymbol{p}_b \trianglelefteq \boldsymbol{p}_a$) if and only if $\boldsymbol{p}_{bi} \leq \boldsymbol{p}_{ai}$ for every $i = 1,2,3$. Profile $\boldsymbol{p}_a$ is (strictly) more unsatisfied than profile $\boldsymbol{p}_b$ (written $\boldsymbol{p}_b \vartriangleleft \boldsymbol{p}_a$) if and only if $\boldsymbol{p}_b \trianglelefteq \boldsymbol{p}_a$ and there is at least one i such that $\boldsymbol{p}_{bi} < \boldsymbol{p}_{ai}$.*

The pair $(P, \trianglelefteq)$ is a *partially ordered set* (or a *poset*, for short); with a little abuse of notation, it will be denoted simply by P. Clearly, not all the profiles in P can be ordered according to Definition 1. For example, $\boldsymbol{p}_a = 132$ and $\boldsymbol{p}_b = 212$ cannot be ordered since $\boldsymbol{p}_{a1} < \boldsymbol{p}_{b1}$, but $\boldsymbol{p}_{b2} < \boldsymbol{p}_{a2}$. The Hasse diagram of P is depicted in Figure 2; each circle represents a profile, explicitly reported inside.

5. Evaluation of subjective well-being

5.1. **Multidimensional evaluation as a comparison problem.** Broadly speaking, when addressing multidimensional evaluation studies, either one may search for an absolute scale against which computing scores or one may follow a "benchmark" approach, anchoring evaluation scores to some kind of threshold, usually given exogenously. Since in socio-economic evaluation there are no absolute scales to draw upon, we maintain that and address the evaluation issue as a comparison problem, where multidimensional profiles have to be assessed against a set of profiles chosen as benchmarks or prototypes. The rest of this section briefly describes how poset theory allows for such multidimensional comparisons. More details on technicalities can be found in (Fattore et al 2011a. Fattore et al. 2011b)

The evaluation function. The fundamental tool for evaluating well-being is the definition of an *evaluation function* f, assigning a satisfaction degree (or, as in our case, an unsatisfaction degree) to any profile in the poset. The evaluation function is defined on the profile poset and, for sake of logical consistency, must enjoy very simple properties, that is:

$$f(\boldsymbol{p}) \geq 0 \qquad (1)$$

$$\boldsymbol{p} \trianglelefteq \boldsymbol{q} \implies f(\boldsymbol{p}) \leq f(\boldsymbol{q}). \qquad (2)$$

[2]Original variables are coded as $v317$, $v318$ and $v323$.

[3]The content of the present paper reflects only author's vision and does not involve any of the cited Institutions.

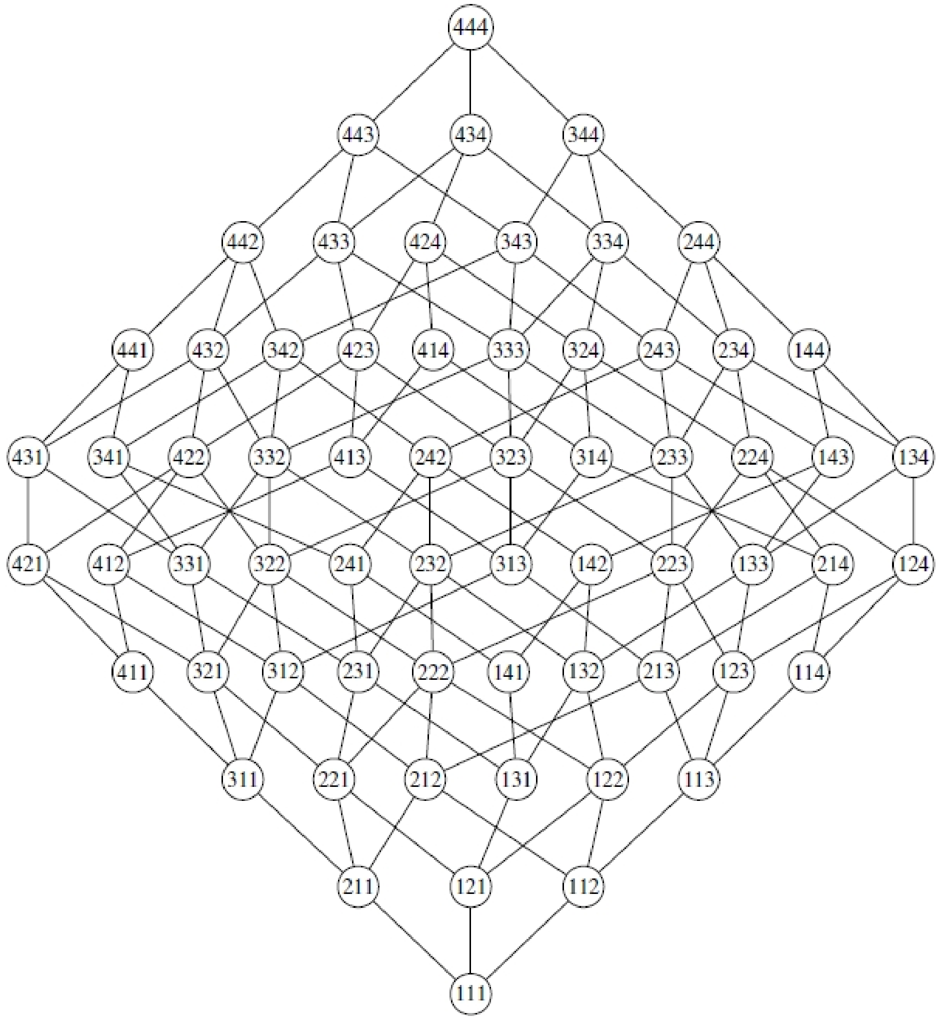

FIGURE 2. Hasse diagram of profile poset P.

where $\boldsymbol{p}$ and $\boldsymbol{q}$ are well-being profiles. As usual in evaluation studies, we add the requirement that the evaluation function scores profiles in the interval $[0, 1]$. Such properties are natural requirements for any meaningful evaluation function and, clearly, do not suffice to define it. The way a suitable evaluation function can be built in practice will in fact be introduced in the following.

5.2. **Comparing multidimensional profiles.** Consider the poset P whose Hasse diagram is depicted in Figure 1. Based on this simple partial order structure, one can only state that a statistical unit sharing profile 444 is maximally unsatisfied (given the evaluation dimension considered in P) and one sharing profile 111 is minimally unsatisfied. Similarly, one can state that sharing profile 232 reveals less subjective well-being than sharing profile 112

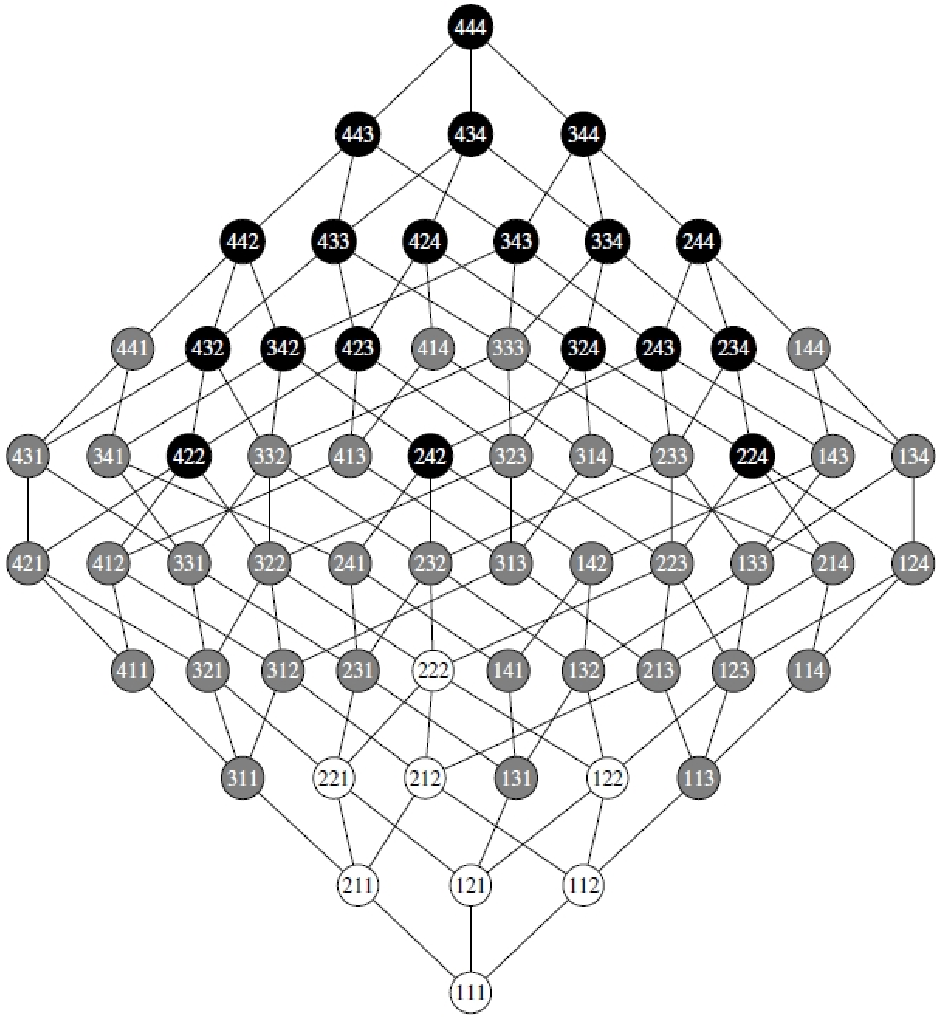

FIGURE 3. Hasse diagram of profile poset P, with subsets U (black nodes), A (gray nodes) and S (white nodes) identified by the threshold selection.

and so on for all comparable pairs of profiles. *A priori*, nothing can be said about incomparable profiles, such as 221 and 113. To enhance the evaluation methodology, some extra information is needed and for this reason a subjective well-being threshold is required.

Thresholds and antichains. As in many evaluation studies, one is primarily interested in identifying statistical units affected by bad socio-economic conditions: the poor, the deprived or the unsatisfied, to make some examples. In our example, this means looking for a threshold identifying the set U of profiles shared by people with an insufficient subjective well-being. Formally stated, the set U comprises all those profiles scored to 1 by the evaluation function:

$$U = \{\boldsymbol{p} \in P : \ f(\boldsymbol{p}) = 1\}. \tag{3}$$

Since $f(\boldsymbol{p})$ is monotonic and $f(\boldsymbol{p}) \leq 1$ for any $\boldsymbol{p} \in P$, if $f(\boldsymbol{p}) = 1$ and $\boldsymbol{p} \trianglelefteq \boldsymbol{q}$, then $f(q) = 1$. In other words, if $\boldsymbol{p} \in U$ and $\boldsymbol{p} \trianglelefteq \boldsymbol{q}$, then $\boldsymbol{q} \in U$, that is, U is an upset of P. As such, there is a unique antichain $\underline{u}$ such that $U = \underline{u}\uparrow$. A profile $\boldsymbol{p}$ is assigned score 1 if and only if it is an element of $\underline{u}$ or if its subjective well-being is worse than the subjective well-being of at least one element of $\underline{u}$. Thus the antichain $\underline{u}$ "separates" profiles of insufficient subjective well-being from the others, as such it is the poset analogous of a threshold in ordinary evaluation studies.

Application to well-being data: the threshold. Let us consider Figure 2. Black nodes represent profiles of insufficient subjective well-being. As can be noticed, all of them belongs to the upset generated by the antichain $(422, 242, 224)$ which in this case represents the thresholds. The three profiles forming the thresholds can be considered as benchmarks against which comparing the subjective well-being of all the other profiles. The choice of profiles 422,242 and 224 as benchmarks is for exemplification purposes. Clearly, the definition of a meaningful threshold is an essential step in evaluation studies, but it is out of the scope of this paper which has mainly a methodological aim. In the unidimensional case, the thresholds distinguishes statistical units into two groups (for example, poor and non-poor). As clear by the Hasse diagram of Figure 2, in a multidimensional ordinal setting this is not the case. In fact, given the threshold, two other important subsets A and S of P can be defined:

$$S = \{\boldsymbol{p} \in P : \boldsymbol{p} \trianglelefteq \boldsymbol{u} \text{ for all } \boldsymbol{u} \in \underline{u}\} \tag{4}$$

$$A = \{\boldsymbol{p} \in P : \exists u \in \underline{\boldsymbol{u}} : \boldsymbol{p} || \boldsymbol{u}\} \tag{5}$$

The set S comprises all the profiles which are below *any* element of the threshold, while the set A comprises all the profiles that are not comparable with at least on element of the threshold. S is the intersection of all the downsets generated by elements of the threshold. Its elements can be unambiguosly considered as representing good socio-economic situations (i.e. sufficient subjective well-being, in our example). As such, they are assigned score equal to 0 by the evaluation function. Differently, elements of A cannot be unambiguously considered as "good"or "bad", since they are incomparable with one or more elements of $\underline{u}$. As a result, the evaluation function will assign scores in $(0, 1)$ to such elements. In summary, sets U, S and A form a partition of P and $f(U) = 1$, $f(S) = 0$, $f(A) \in (0, 1)$. Referring again to Figure 2, elements of S are represented as white nodes, while elements of A are depicted as gray nodes. This shows how introducing a threshold differentiates among nodes, based on their different positions in the Hasse diagram *with respect to the threshold*.

Computation of the evaluation function. Here, we briefly describe how the evaluation function can be defined, based on poset theory. For technical details, see Fattore et al (2011a,b). The basic result. already stated, is that *any* partial order can be uniquely described as the intersection of a maximal set of linear orders, namely the set of its *linear extensions*. If the original poset would be a linear order, the classification of profiles in unsatisfied and satisfied would be trivial: if a profile is ranked above an element of the threshold, then it is to be scored as unsatisfied; if it is ranked below *all* the elements of the threshold, then it is to be scored as satisfied. So the basic idea is to list all the linear extension of the profile poset, counting the relative frequency by which a profile is scored as unsatisfied or satisfied in the set of linear extension. Since listing all the linear extension of a partial order like that discussed in this paper is virtually impossible, the computation of the evaluation function is usually performed based on a sample of linear extensions.

Application to well-being data: some results. The procedure outlined above has been applied to the well-being data pertaining to Italy and previously described. The evaluation function has been computed over a sample of 10^9 linear extensions and individuals have been assigned the score of the profile they share. The overall results are shown in Table 1. We have computed the mean unsatisfaction level (μ_1) for the entire population and for people sharing profiles with a strictly positive unsatisfaction degree (μ_2). While μ_1 can be considered as a (fuzzy) extension of the classical Head Count Ratio, μ_2 reveals something about the level of unsatisfaction of people suffering from some form of unsatisfaction. In a sense, μ_2 can be considered as a proxy of the level of unsatisfaction of not completely satisfied people. All the results are provided at regional level and for males and females. Results depend heavily on the choice of the threshold. Any interpretation of the results must then be considered with great care, since in this paper the threshold has been chosen for illustrative purposes only. Anyway, the different status of people from the North and the South of Italy clearly emerges, both in terms of μ_1 and μ_2. Trentino - Alto Adige emerges has the region where people is more satisfied and this is consistent with the well-known administrative efficiency and the subsidiary organization of that region. It is also remarkable that females show systematically higher unsatisfaction scores than males. In particular, the male-female difference in indicator μ_2 is particularly clear and increases in the Southern regions.

6. CONCLUSIONS

In this paper, we have outlined and applied to real data a new methodology for evaluating social traits, like satisfaction, poverty, well-being, based on ordinal data. The methodology relies entirely on the ordinal nature of the data and avoids any aggregation of evaluation variables, avoiding the need for scaling ordinal scores into cardinal figures. The final scores are in fact computed exploiting the partial order structure of the data. Although the principal aim of the paper is methodological, the proposed application to subjective well-being data gives the flavour of what can be obtained by applying the methodology to this kind of studies and shows that it provides a new, consistent and effective tool for social evaluation.

References

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	μ_1	μ_2	μ_1 - male	μ_2 - male	μ_1 - female	μ_2 - female
Piemonte-VdA.	0.12	0.20	0.12	0.19	0.13	0.22
Lombardia	0.12	0.20	0.11	0.19	0.13	0.21
Trentino - AA.	0.07	0.10	0.07	0.10	0.08	0.10
Veneto	0.12	0.20	0.12	0.19	0.13	0.21
Friuli-VG.	0.11	0.22	0.09	0.22	0.12	0.21
Liguria	0.11	0.18	0.09	0.16	0.12	0.19
Emilia-Romagna	0.11	0.20	0.10	0.18	0.12	0.22
Toscana	0.10	0.22	0.07	0.18	0.11	0.24
Umbria	0.13	0.21	0.10	0.17	0.16	0.24
Marche	0.13	0.21	0.12	0.21	0.13	0.21
Lazio	0.15	0.24	0.12	0.21	0.17	0.27
Abruzzo	0.13	0.24	0.12	0.20	0.14	0.27
Molise	0.15	0.26	0.15	0.26	0.15	0.26
Campania	0.21	0.32	0.15	0.22	0.24	0.39
Puglia	0.18	0.28	0.13	0.21	0.21	0.32
Basilicata	0.15	0.28	0.14	0.25	0.17	0.30
Calabria	0.14	0.29	0.12	0.23	0.15	0.34
Sicilia	0.16	0.32	0.13	0.27	0.18	0.35
Sardegna	0.11	0.34	0.09	0.33	0.13	0.35
ITALIA	0.13	0.23	0.11	0.20	0.15	0.26

TABLE 1. Unsatisfaction degrees at national and regional level, for males and females, year 2010