Reconciliation of Time Series according to a Growth Rates Preservation Principle

Tommaso Di Fonzo and Marco Marini

Abstract The reconciliation of a system of time series is known in the literature as the statistical process of adjusting preliminary values of the series to satisfy both temporal and contemporaneous constraints. In this paper we propose new reconciliation procedures based on the Growth Rates Preservation (GRP) principle, which explicitly preserves the period-to-period growth rates of the preliminary series. A non-linear constrained minimization problem is solved through a Newton's optimization method, which exploits the analytical gradient and Hessian of the GRP objective function. We apply these procedures to two real-life applications and compare them with the state-of-the-art reconciliation procedures. The results show that simultaneous and two-step reconciliation procedures based on the GRP criterion are worthy candidates in terms of both quality of results and computational time, even for large systems with many constraints.

Key words: Benchmarking, Reconciliation, Growth rates preservation, Newton's method.

1 Introduction

A system of time series is subject to an adjustment process when there are required restrictions on the values of the series that are not readily satisfied. We deal with two types of restrictions: (i) *temporal aggregation* constraints, which require that the high-frequency adjusted series be in line with known, more reliable low-frequency aggregates, and (ii) *contemporaneous* constraints, assuming the form of linear combinations of the variables which should be fulfilled in every observed period. These constraints are typical of socio-economic systems of time series frequently handled by statistical agencies. Typical examples are the compilation of national accounts statistics and the seasonal adjustment of component and aggregate time series through a direct approach.

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The reconciliation of a system of time series is the statistical process that aims at restoring consistency with respect to both types of constraints. This adjustment should be done according to some movement preservation principle, such that the temporal profiles of the original series are preserved to the highest possible degree. In this paper we propose simultaneous and two-step reconciliation procedures based on the Growth Rates Preservation (GRP) principle, as expressed by Causey and Trager (1981). This criterion is explicitly related to the growth rate of the involved variables, which is a natural measure of the movement of a time series.

2 Simultaneous and two-step reconciliation procedures based on the GRP principle

For the temporal benchmarking of a single series, Causey and Trager (1981) propose to solve the constrained minimization problem

$$\min_{y_t} \sum_{t=2}^{n} \left(\frac{y_t}{y_{t-1}} - \frac{p_t}{p_{t-1}} \right)^2 \qquad \text{subject to} \quad \sum_{t \in T} y_t = Y_T, \quad T = 1, \ldots, N \tag{1}$$

where y_t is the unknown target variable, p_t is the preliminary series, Y_T is the low frequency temporal benchmark, and N and n are the number of low-frequency (say, annual) and high-frequency (say, monthly or quarterly) observations, respectively.

In this paper we extend the univariate GRP criterion in (1) to a system of M (> 1) variables subject to both temporal and contemporaneous constraints:

$$\min_{y_{j,t}} \sum_{j=1}^{M} \sum_{t=2}^{n} \left(\frac{y_{j,t}}{y_{j,t-1}} - \frac{p_{j,t}}{p_{j,t-1}} \right)^2 \text{ subject to } \begin{cases} \sum_{t \in T} y_{j,t} = Y_{j,T}, & T = 1, \ldots, N \\ \sum_{j} c_j^h y_{j,t} = z_t^h, & h = 1, \ldots, K \end{cases} \tag{2}$$

where K denotes the number of contemporaneous constraints. The constants c_j^h are known, real-valued constants that define the accounting restrictions across the variables (e.g. a sequence of ones on regional data), whereas z^h are the high-frequency known quantities that are associated with those restrictions (e.g. the national total).

The constrained minimization problem (1) does not have linear first–order conditions for a stationary point, and thus it is not possible to find an explicit, analytic expression for the solution. In addition, the GRP criterion in (1) is a non-linear and non-convex function of the unknown variables (Di Fonzo and Marini, 2011b). A local solution can be found through several iterative minimization algorithms (Nocedal and Wright, 2006). Di Fonzo and Marini (2011b) transform the original constrained problem into an unconstrained one, and then develop a Newton's method which exploits the analytical expressions of the gradient and the Hessian of the objective function.

We follow the same approach to solve the reconciliation problem (2): (i) the constrained problem is transformed into an unconstrained one[1], and (ii) the Newton's method is applied to find a minimum in the range-space of the transformed variables. The reconciliation problem is therefore solved simultaneously, with both temporal and contemporaneous constraints considered at the same time.

A simultaneous reconciliation procedure has already been proposed by Di Fonzo and Marini (2011a). It is based on the modified Proportional First Differences (PFD) criterion, which is considered a good approximation of the GRP principle, and is the natural extension of the classical temporal benchmarking procedure by Denton (1971). The main advantage of the Denton's approach is that the benchmarked/reconciled series can be expressed in closed form, according to simple matrix formulae. Di Fonzo and Marini (2011a) consider also *two-step* reconciliation procedures, that are computationally less demanding if sparse matrices computation facilities are not used. The first step consists in a univariate benchmarking to restore the temporal consistency of every series in the system; in the second step, the series are reconciled one low-frequency (say, year) at a time by using a least squares balancing procedure. Two different criteria were considered in the second step: a proportional adjustment (BB), and an adjustment based on the square of the preliminary (benchmarked) value (ST)[2].

This paper primarily aims to assess the performance of the simultaneous reconciliation procedure based on the GRP criterion (Sim GRP), comparing it with the simultaneous reconciliation procedure based on the modified PFD criterion (Sim MD). Also, we are interested in assessing two-step reconciliation procedures that use, at the first step, the GRP univariate benchmarking procedure developed by Di Fonzo and Marini (2011b). The combination of two options at the first step (GRP and MD) and the two options used in Di Fonzo and Marini (2011a) at the second step (BB and ST) gives rise to four alternative two-step reconciliation procedures (MD-BB, GRP-BB, MD-ST and GRP-ST). The next section will present a comparison of the six procedures based on two real-life applications.

3 Applications

We consider two systems of time series: the EUQSA system, 175 quarterly variables of the European Union's quarterly national accounts, and the MRTS system, 236 monthly series of Canadian seasonally adjusted retail trade by provinces (for more details, see Di Fonzo and Marini; 2011a).

In order to assess the performance of the procedures, for each series we calculate the Mean Absolute Adjustment (*MAA*) to the percentage growth rates, that is

[1] For a reconciliation problem, an explicit QR factorization of the transposed matrix of constraints, as shown in Di Fonzo and Marini (2011b) for a single series GRP benchmarking, is rather complicated. In fact, that matrix does not have full rank and its sparsity structure is not as simple as in the case of univariate benchmarking. However, a less efficient but effective numerical sparse QR factorization can still be used to generate the null space matrix.

[2] For a statistical interpretation in terms of reliability of the preliminary series, see Di Fonzo and Marini (2011a).

$$MAA_j = 100 \times \frac{1}{n-1} \sum_{t=2}^{n} \left| r_{j,t}^{y} - r_{j,t}^{p} \right|, \quad j = 1, \ldots, M$$

where $r_{j,t}^{y} = \left(y_{j,t} - y_{j,t-1}\right)/y_{j,t-1}$ and $r_{j,t}^{p} = \left(p_{j,t} - p_{j,t-1}\right)/p_{j,t-1}$ are the growth rates of the reconciled and preliminary series, respectively. Overall indices for the whole system of time series are calculated accordingly.

Table 1 shows summary statistics on the *MAA* values for the two systems. Sim GRP outperforms the other procedures in both systems, with the lowest mean, median and standard deviation. Sim GRP is also the procedure with the highest number of series with minimum *MAA* value (41.1% and 29.2%). As expected, we note that on average Sim MD is very close to Sim GRP. However, Sim MD tends to generate higher adjustments to series with sudden breaks in their levels (e.g. almost 25% to a small EUQSA series)[3].

Concerning the two-step procedures, we note that (i) using the GRP benchmarking at the first step improves upon using the Denton PFD benchmarking procedure (especially when ST is used in the second step), and (ii) the GRP-ST reconciliation procedure stands out as a valid alternative of the Sim GRP procedure.

Table 1 Summary statistics on *MAA* for the EUQSA and the MRTS systems

	Two-step				**Simultaneous**	
	MD-BB	GRP-BB	MD-ST	GRP-ST	Sim MD	Sim GRP
EUQSA system						
Mean	3.0949	2.9787	1.2717	1.1097	1.2799	1.0883
Median	2.1178	2.1635	0.3874	0.3884	0.4212	0.3780
St. dev.	3.2645	2.9854	2.3604	1.4987	2.3671	1.4641
Max	27.2699	29.1887	24.3963	6.9151	24.4550	6.3662
% Min	8.6	10.9	5.7	20.6	13.1	41.1
MRTS system						
Mean	1.5312	1.5315	0.7064	0.7030	0.7054	0.6997
Median	1.4222	1.4214	0.4708	0.4601	0.4874	0.4555
St. dev.	0.7417	0.7484	0.6988	0.7021	0.6957	0.6928
Max	5.2620	5.3978	4.3494	4.3495	4.3538	4.3065
% Min	10.2	9.3	5.9	20.8	24.6	29.2

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[3] A deeper analysis of the results, with more details and graphical evidences, will be presented in the extended version of the paper.